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Rainfall - Prediction of cyclic changes

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ABSTRACT

As rainfall is beyond human intervention, it requires accurate forecast, which is yet to be solved. This attempt exhibits a method to forecast rainfall. Periodic oscillations in the rainfall were observed in the scatter diagram and therefore Harmonic Analysis was used. To justify the validation of the prediction, Markov-transition probability matrix and its powers are used.

Key words: Harmonic Analysis, Markov transition probability, rainfall, forecasting and stationarity.

The most important climatic factor for agricultural product is the amount of rainfall, which is highly unpredictable and erratic in nature. As an agricultural country, Indian economy depends significantly on agricultural production. Due to the nonavailability of the exact onset of monsoon its intensity and duration, the farmers were unable to predict the exact time of operation. Hence it was thought that a better forecast could bring cheers to the farming community. Hence this attempt is made to forecast the possible rainfall with more amount of reliability, for Sulur in Tamilnadu, which is subject to frequent occurrence of drought. This result will help the planners in decision making with regard to agriculture.

MATERIALS AND METHODS

Data on the monthly rainfall for fifty years from 1951 to 2000 collected from the basic records maintained in the observatory in Sulur when exhibited in the scatter diagram (Fig.1) showed periodic oscillations and therefore Harmonic Analysis was used.

The general form of the Fourier series is

$$y = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos znt + b_n \sin znt\right)$$

where

y	=	Monthly rainfall
t	=	time point
a_n , b_n	=	Fourier coefficients
n	=	number of harmonic
z	=	$360^0/12 = 30^0$

Similarly the Markov-chain transition probability analysis was carried out. Symbolically stochastic process is defined as any family of random variables $\{x_t, t \in T\}$,

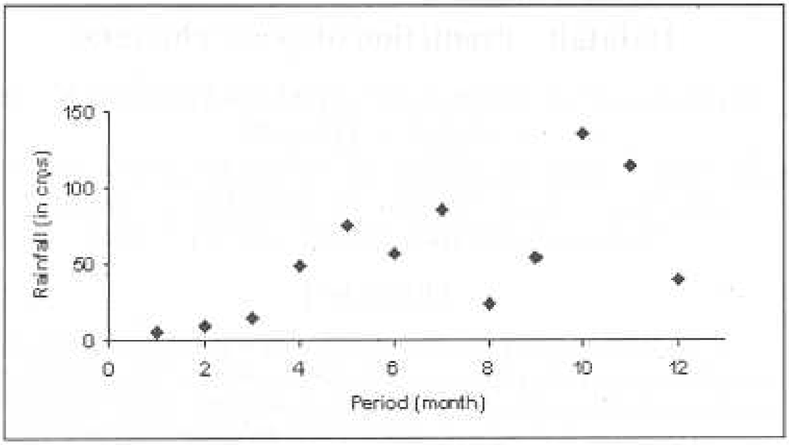

Fig. 1 : Scatter diagram showing periodic variations of rainfall

here x_t is the observation at time t and T is the time range. For example x_1 stands for rainfall in January.

The stochastic process $\{X_n, n = 0,1,2, ...\}$ is called a Markov chain if for

$j,k, j_1, ... j_{n-1} \in N$ (or any subset of T)

$\Pr\{X_n=k / X_{n-1} = j, X_{n-2} = j_1, ...X_0 = j_{n-1}\}$
$= \Pr\{X_n=k / X_{n-1} = j\} = p_{jk}$

whenever the first member is defined.

That is the probability of any random variable assuming a particular value in any time will depend only on the just previous period. For example, today's rainfall should depend only on yesterday's and not on the preceding or succeeding day's rainfall. i and j are called states and p_{jk}'s are called transition probabilities.

The transition probabilities p_{jk} satisfy $p_{jk} \geq 0$ and $\Sigma p_{jk} = 1$ for all j.

These probabilities may be written in the matrix form

$$P = \begin{pmatrix} p_{11} & p_{12} & p_{13}\cdots\cdots \\ p_{21} & p_{22} & p_{23}\cdots\cdots \\ \cdots\cdots & & \\ \cdots\cdots & & \end{pmatrix}$$

This is the transition probability matrix or matrix of transition probabilities of a Markov chain.

RESULTS AND DISCUSSION

Harmonic analysis

The estimated value of the Fourier coefficient for Sulur are

$a_0 = 57.7$

$a_1 = 1.565$	$b_1 = 1.265$
$a_2 = -1.395$	$b_2 = -1.1$
$a_3 = 1.178$	$b_3 = 0.925$
$a_4 = -1.006$	$b_4 = 0.617$
$a_5 = 0.875$	$b_5 = 0.486$
$a_6 = -0.663$	$b_6 = -0.213$

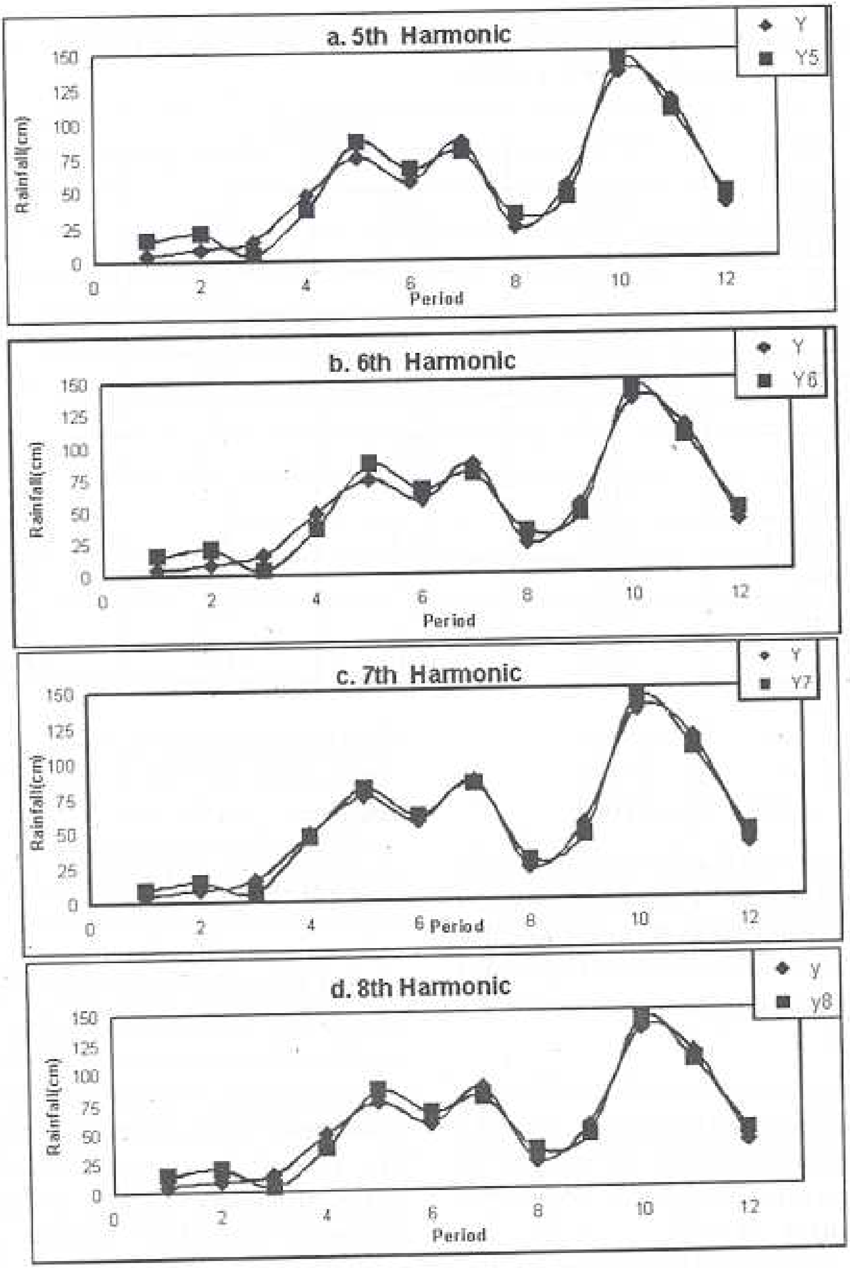

Fig. 2 (a – d) : Comparison of observed and estimated rainfall for 5[th], 6[th], 7[th] and 8[th] Harmonic

Table 1: Absolute deviations for observed and estimated rainfall by different harmonics from 1951 to 2000

Month	Harmonics			
	5th	6th	7th	8th
January	11.0	10.3	8.9	9.1
February	11.4	12.2	8.9	11.0
March	10.7	10.5	9.3	10.3
April	10.2	9.8	9.6	9.6
May	10.2	10.0	9.6	9.9
June	11.3	10.3	7.9	9.4
July	9.3	8.2	7.1	8.0
August	10.5	9.5	8.9	9.1
September	11.4	10.4	9.0	10.1
October	9.2	8.1	7.1	8.0
November	11.3	9.4	7.9	8.8
December	11.2	11.0	8.9	10.8

$a_7 = 0.436$ $b_7 = 0.083$
$a_8 = 0.13$ $b_8 = 0.046$
$a_9 = -0.056$ $b_9 = -0.018$

The measured and expected rainfalls are presented in Fig (2), using the respective harmonics. It is observed that difference in the observed and estimated rainfall levels are higher on the lower order harmonics.

Using these harmonic coefficients rainfall was predicted at the end. Here the criteria for deciding on the number of harmonics is made as follows. The Fourier coefficients ai, i = 0,1,2,.....n; bj, j = 1,2,....n were estimated based on the formula given in materials and methods. The rainfall predictions were computed successively for each harmonic levels. For each harmonic the expected value obtained through the formula were compared for each year with the observed. In the initial level the difference between the observed and expected values were higher and started reducing at higher harmonic levels. After a particular stage this difference started increasing again. The calculation is stopped as soon as this turning point is reached. In the present study as per the computed values shown in Table 1 the absolute deviations started decreasing after the seventh harmonic and hence the estimation was stopped at the eighth harmonic. Since the differences are much higher in the lower harmonics, they were not presented in the table. The absolute deviations for observed and estimated are presented in Table 1.

Table 2: Chi-square coefficients between observed and estimated rainfalls for different Harmonics

Month	Harmonics			
	5th	6th	7th	8th
January	10.8	10.4	8.4	9.7
February	13.0	11.8	10.4	11.0
March	12.0	10.8	9.9	10.6
April	12.3	11.6	10.2	11.1
May	11.1	10.8	9.7	10.0
June	9.3	8.8	7.7	8.4
July	9.0	8.2	7.5	7.7
August	9.6	9.0	8.1	8.8
September	9.5	8.9	8.2	8.6
October	8.6	8.1	7.2	7.7
November	9.0	8.4	7.5	7.9
December	8.9	8.8	8.0	8.5

Results presented in Table 1 reveal that the absolute deviations are minimum only during the months of October, July, November and June in all the harmonics and is maximum during the month of February in all the harmonics.

Deviations were tested through chi-square and the estimated chi-square are presented below in Table 2

The minimum chi-square is for October (Table 2) followed by July, November and June. For all other months the results showed higher deviations. Here also chi-square is maximum for February. This might probably be due to the fact that regular rainfall occurred in this station in all the above four months and in the other months there is no consistency in rainfall occurrence. Moreover January is the least average rainfall month and November is the highest average rainfall month as indicated in the Table 3. Thus in Harmonic Analysis, adjustment of harmonic number help in reducing absolute deviation between the observed and the expected value.

This is again ascertained by the minimum coefficients of variation values in the regular rainfall months and the higher coefficient of variability in the non-seasonal months, the maximum being in February.The mean, standard deviation, coefficient of variation, skewness and kurtosis are presented below in Table 3.

The results presented in Table 3 reveal that the standard deviation is the highest for the month of May, the period of summer rainfall. This might be due to the unpredicted nature of the summer rainfall

Table 3: Mean, Standard deviation, Coefficient of variation, Skewness and Kurtosis

Month	Mean	Standard Deviation	Coefficient of variation	Skewness	Kurtosis
January	4.9	10.132	208.0	2.343	5.125
February	8.7	20.347	232.9	2.662	6.685
March	15.0	32.026	214.1	3.038	9.648
April	47.9	48.588	101.4	1.318	1.509
May	114.9	113.946	99.2	2.04	4.778
June	74.4	**62.016**	83.4	0.975	0.548
July	85.8	50.998	59.4	1.206	1.646
August	23.3	28.394	122.1	1.174	0.103
September	53.8	66.702	124.0	1.59	2.094
October	56.7	32.153	56.7	1.314	2.509
November	134.9	98.085	72.7	1.793	4.107
December	39.5	58.775	148.9	2.096	4.381

during the period of study. The positive skewness indicates more number of years with higher rainfall and this is justified by the kurtosis value. Again the standard deviation is least during January. Here the average rainfall is low. This may be the cause for the low level of standard deviation. In the regular rainfall months namely June, July, October and November the skewness and kurtosis are very low as seen in the Table 3. This might be due to the consistency of rainfall in these months.In Table 4 the predictions for rainfall for any year is presented

In the Harmonic Analysis the average rainfall over the fifty years for each month was used. Hence the expected value got through the Harmonic Analysis will only give the expected average rainfall for any year. It can not be used for predicting the exact rainfall for any desired year in the selected month. The advantage is that the result with the standard deviation will give the most likely rainfall for any year, and

Table 4: Prediction of rainfall with standard error for any year after 2000

Months	**Predictions**
January	4.9 ± 10.1
February	8.7 ± 20.7
March	15.0 ± 32.0
April	47.9 ± 48.6
May	114.9 ± 113.9
June	74.4 ± 62.0
July	85.8 ± 51.0
August	23.3 ± 28.4
September	53.8 ± 66.7
October	56.7 ± 32.2
November	134.9 ± 98.1
December	39.5 ± 58.8

the disadvantage is that no exact value is available for any future period.

Markov-chain Transition Probability Analysis

In this study the Markov chain probability analysis is attempted since, there is chance for moving the rainfall from one state (level) to another state. For example, change of rainfall from the state 1-50 to the state 51-100 or any other interval (state) in the same period. The above definitions were used to work out the transition probabilities. For example the transition from state 1 to state 2 that is, from 1-50 cm rainfall to 51-100 cm rainfall is obtained by the number of years in which the rainfall has increased from

1-50 to 51-100 divided by 50. In this way all transition probabilities were calculated.

Let the Markov transition probabilities be : P_{-1} for the month May to June: P_2 for June to July; P_3 for July to August; P_4 for September to October; P_5 for October to November and P_6 for November to December

Transition probabilities were considered only for the above six periods, since in the other months namely January to May, August, September, December, most of the transitions will be zero due to no rains. For uniformity the transition intervals were taken as 1 – 50 cm, 51 – 100 cm, 101 – 150 cm, ... and 351 – 400 cm. If in a period the maximum rainfall so far has not crossed 250 cm it will have only 5 states and the corresponding matrix will be 5x5.

$$P_1 = \begin{pmatrix} 0.826 & 0.174 \\ 0.926 & 0.074 \end{pmatrix}$$

$$P_2 = \begin{pmatrix} 0.930 & 0.070 \\ 1.000 & 0.000 \end{pmatrix}$$

$$P_3 = \begin{pmatrix} 0.809 & 0.191 \\ 1.000 & 0.000 \end{pmatrix}$$

$$P_4 = \begin{pmatrix} 0.147 & 0.206 & 0.353 & 0.206 & 0.088 \\ 0.286 & 0.286 & 0.143 & 0.000 & 0.286 \\ 0.400 & 0.000 & 0.400 & 0.000 & 0.200 \\ 0.500 & 0.000 & 0.000 & 0.000 & 0.500 \\ 0.000 & 1.000 & 0.000 & 0.000 & 0.000 \end{pmatrix}$$

$$P_5 = \begin{pmatrix} 0.500 & 0.300 & 0.100 & 0.000 & 0.100 & 0.000 \\ 0.300 & 0.600 & 0.000 & 0.100 & 0.000 & 0.000 \\ 0.529 & 0.235 & 0.118 & 0.059 & 0.059 & 0.000 \\ 0.429 & 0.000 & 0.000 & 0.143 & 0.143 & 0.286 \\ 0.000 & 0.500 & 0.000 & 0.000 & 0.000 & 0.500 \\ 0.500 & 0.167 & 0.167 & 0.000 & 0.167 & 0.000 \end{pmatrix}$$

$$P_6 = \begin{pmatrix} 0.800 & 0.040 & 0.040 & 0.040 & 0.080 \\ 0.600 & 0.200 & 0.200 & 0.000 & 0.000 \\ 1.000 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.667 & 0.000 & 0.000 & 0.333 & 0.000 \\ 0.857 & 0.000 & 0.143 & 0.000 & 0.000 \end{pmatrix}$$

In order to know the stationarity, various powers of these matrices were constructed and they are represented respectively by P_i^2, P_i^3, P_i^4,.......... i = 1,2,3,....6 Since the probability value repeats itself after the seventh power, the computation was stopped at the eighth power. That is the difference between P^7 and any higher power of P is close to zero. The first three decimals in all the differences are zero and in certain cases the difference is exactly zero. Thus it can be concluded that stationarity occurs in

Markov – transition probabilities at the seventh period for Sulur. Comparing the results obtained in Harmonic Analysis and Markov chain transition probability analysis, the number seven has a role in the rainfall, which might be the cyclic period for drastic changes.

A rare observation noticed here is that transition from either 2 or 1 to 1 are same similarly transition from 2 or 1 to 2 are also equal indicating that two states are almost identical in nature in the seventh year. This indicates either equal rain throughout or drought. Similarly non-stationarity indicates most irregular and unpredictive rainfall seasons. Historical data also shows that the period of severe drought occurs once in seven years and the period of excess also occurs once in seven years. Thus the Markov-chain transition probability analysis gives a clue for the probable drought occurrence.

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