Rainfall intensity-duration-return period equations and nomographs for Andhra Pradesh

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ABSTRACT

Rainfall intensity-duration-return period (IDRP) equations and nomographs were developed earlier in 1979 for two stations in Andhra Pradesh (Hyderabad and Vishakhapatnam) due to non-availability of recording rainfall data of more stations. In the present study the IDRP equations and the nomographs for 22 stations of the state have been developed based on long period data obtained from the IMD, Pune. The results are discussed in relation to the general equation and nomograph for the Andhra Pradesh state.

Key words: Rainfall intensity, Return period, Recurrence interval, Nomograph, Run-off disposal.

The Intensity, duration and frequency of rainstorms play a vital role in determining the rate of soil erosion. Intense rainfall activities with greater kinetic energy dislodge soil particles and splashes them in suspension in runoff. Among other factors, the amount of runoff is determined by rainfall intensity, duration and amount. A rainfall of longer duration reduces the infiltration capacity of soil. As a result, a long duration rainstorm produces considerable runoff regardless of its intensity. The capacity of a runoff conveyance system is usually based on a certain depth of rainfall to be expected during a specified period of time. Farm terraces, culverts, bridges and flood control structures are thus designed.

The significance of rainfall intensity, duration and frequency analysis is also important from economic considerations. An over designed structure involves excessive cost while under designed structure would be unsafe and also involves high recurrence expenditure on maintenance and replacement etc. An intermediate design would provide a structure with reasonable initial and maintenance costs.

Rainfall intensity-duration-return period (IDRP) equations and nomographs have been developed earlier only for a few stations (Gupta et al., 1968; Raghunath et al. 1969; Khullar et al., 1975; Senapati et al., 1976; and Ram babu et al., 1980). No attempts have been made in the past to develop such a tool for a state as a whole.

In the present study, based on observed data for 22 stations, the intensity-duration-return period equations and nomographs for individual stations as well as general equation and nomograph for the state of Andhra Pradesh have been developed and discussed.
MATERIALS AND METHODS

Data

To derive prediction equations for intensity-duration-frequency and for development of nomographs, the continuous recorded rainfall data for 22 stations situated in the state of Andhra Pradesh were obtained from India Meteorological Department, Pune. Due to the non-availability of data for long period for all the stations under study, more than 16 years records have been used for various duration varying from 1974 to 1995. The locations of these stations are shown in Fig.1. The data for all stations was tested for reliability using the procedure of Ogrosky and Mockus (1957) and it was observed that the length of record for all the stations are adequate and hence could be used for frequency analysis.

Frequency analysis and development of frequency lines

Various methods have been proposed for frequency analysis and there are several theoretical interpretations or reasoning for the preference of one method or the other (Chow, 1964). Mathematical or graphical methods are generally used for frequency analysis. When the records are of short duration, the sampling error would be large. A rigid mathematical treatment is not justified when the data are available for less than 30 years (Dalrymple, 1960). The present data used for analysis being of short period ranging between 16 to 20 years, graphical methods have been employed. Gumbel extreme value technique was applied for computation of return periods and the frequency lines were plotted after computing the plotted points by computed method suggested by Ogrosky and Mockus (1957). Frequency lines for 15, 30 minutes, 1 hour, 3, 6, 12 and 24 hours intensity data were developed and plotted on log-normal probability paper (Fig. 2).

Equation for IDRП

Various formulae have been in use for connecting the three parameters - rainfall intensity, duration and return period (Linsley et al., 1949; Schwab et al., 1955; Skurlow, 1960; Nemec, 1973; Gupta et al., 1968; Raghunath et al., 1969; Khullar et al., 1975; Senapati et al., 1976 and Ram Babu et al., 1980). The formula is of the general form:

\[ I = KT^b / (t+b)^d \]  \[ \text{--- (1)} \]

where, \( I \) = intensity of rainfall (cm h\(^{-1}\)), \( T \) = return period (years), \( t \) = duration (hours); \( K, b, a \) and \( d \) are the constants.

Equation (1) was used for developing intensity-duration-frequency relationships. Looking into simplicity in use and quickness, the nomographs were developed for field workers. The detailed methodology for developing IDRП equations and nomographs has been dealt by Ram Babu et al. (1980).

The IDRП equation, (Eq. 1) can be expressed by taking logarithms on both sides as:

\[ \log I = \log K + a \log T - d \log (t+b) \]  \[ \text{--- (2)} \]

\[ \log I = \log K_d - d \log (t+b) \]  \[ \text{--- (3)} \]

where, \( \log K_d = \log K + a \log T \)  \[ \text{--- (4)} \]

The coefficients a, b, d and K were
Fig. 1: Map showing the location of recording rain-guage stations in Andhra Pradesh.

Fig. 2: Frequency distribution of rainfall intensities for various durations - Khammam.
### Table 1: Intensity-duration-return period relationships for various stations in Andhra Pradesh.

<table>
<thead>
<tr>
<th>Station</th>
<th>Equation</th>
<th>Station</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Anantpur</td>
<td>$I = \frac{10.4219 \ T^{0.1995}}{(t + 1.05)^{1.2224}}$</td>
<td>Agogyavaram</td>
<td>$I = \frac{7.2310 \ T^{0.1504}}{(t + 0.80)^{1.1545}}$</td>
</tr>
<tr>
<td>Bapatla</td>
<td>$I = \frac{8.7557 \ T^{0.2004}}{(t + 0.60)^{1.1123}}$</td>
<td>Cuddapah</td>
<td>$I = \frac{4.9890 \ T^{0.1416}}{(t + 0.40)^{0.9855}}$</td>
</tr>
<tr>
<td>Gannavaram</td>
<td>$I = \frac{6.4318 \ T^{0.1110}}{(t + 0.50)^{1.0969}}$</td>
<td>Hakimpet</td>
<td>$I = \frac{9.6860 \ T^{0.2181}}{(t + 1.10)^{1.1010}}$</td>
</tr>
<tr>
<td>Hanamkonda</td>
<td>$I = \frac{9.5064 \ T^{0.1172}}{(t + 0.80)^{1.1779}}$</td>
<td>Kakinada</td>
<td>$I = \frac{10.4721 \ T^{0.1832}}{(t + 1.20)^{0.0970}}$</td>
</tr>
<tr>
<td>Kalingapatnam</td>
<td>$I = \frac{12.8352 \ T^{0.1861}}{(t + 1.40)^{1.1775}}$</td>
<td>Khamman</td>
<td>$I = \frac{7.1024 \ T^{0.1629}}{(t + 0.70)^{1.0551}}$</td>
</tr>
<tr>
<td>Kurnool</td>
<td>$I = \frac{12.2351 \ T^{0.1623}}{(t + 1.30)^{1.1971}}$</td>
<td>Mahbubnagar</td>
<td>$I = \frac{5.3621 \ T^{0.1908}}{(t + 0.60)^{0.9531}}$</td>
</tr>
<tr>
<td>Musulipatnam</td>
<td>$I = \frac{9.5496T^{0.1665}}{(t + 1.10)^{0.6446}}$</td>
<td>Nallorre</td>
<td>$I = \frac{9.8231 \ T^{0.1995}}{(t + 1.40)^{1.0662}}$</td>
</tr>
<tr>
<td>Nandyal</td>
<td>$I = \frac{6.6536 \ T^{0.1546}}{(t + 0.90)^{1.1442}}$</td>
<td>Nidadavole</td>
<td>$I = \frac{8.4807 \ T^{0.1686}}{(t + 0.70)^{0.9907}}$</td>
</tr>
<tr>
<td>Nizamabad</td>
<td>$I = \frac{10.8869 \ T^{0.1722}}{(t + 1.60)^{1.1667}}$</td>
<td>Ongole</td>
<td>$I = \frac{10.4751 \ T^{0.2756}}{(t + 1.10)^{1.1140}}$</td>
</tr>
<tr>
<td>Ramagundum</td>
<td>$I = \frac{9.0769 \ T^{0.1700}}{(t + 0.75)^{1.1483}}$</td>
<td>Retachintala</td>
<td>$I = \frac{10.7574 \ T^{0.1479}}{(t + 1.20)^{1.1676}}$</td>
</tr>
<tr>
<td>Venukonda</td>
<td>$I = \frac{10.1166 \ T^{0.1757}}{(t + 1.05)^{1.1791}}$</td>
<td>Vishakhapatnam</td>
<td>$I = \frac{10.0300 \ T^{0.1628}}{(t + 0.90)^{1.0812}}$</td>
</tr>
<tr>
<td>Andhra Pradesh</td>
<td>$I = \frac{9.0341 \ T^{0.1828}}{(t + 0.95)^{1.1113}}$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

($I =$ intensity (cm/hr); $T =$ return period (year); and $t =$ duration (hour))
evaluated using the seven steps given by Ram Babu et al. (1980) (Appendix I).

Development of nomograph

A nomograph is an alignment that consists of a set of parallel scales, which are suitably graduated. In the present study, there were only three variables and thus the alignment chart had three parallel scales so graduated that a line which joins values on two scales will intersect the third scale at a value which satisfies the given equation. In order to design alignment charts prepared between the variable T, t and I using the procedure suggested by Luzadder (1964) and Ram Babu et al. (1980) for equation of the form \( f_1(u) + f_2(v) = f_3(w) \), the following are required.

(a) the graduation of scales marked with the values of the variable and on which the distances to the graduations are laid off in proportion to the corresponding values of the function of the variables, and

(b) the determination of spacing of the parallel scale. The scale equation for determining functional modulus \( m \), commonly defined as proportionality multiplier used to bring a range of values of particular function with a selected length for a scale, is given as:

\[
m = \frac{L}{f(u_2) - f(u_1)}
\]  --- (5)

where, \( m \) = calculated functional modulus, \( L \) = length of the scale chosen, \( f(u) \) and \( f(u) \) = lower and upper limit respectively of the function.

The unknown functional modulus \( m_w \) was calculated by

\[
m_w = m_u m_v / (m_u - m_v)
\]  --- (6)

where, \( m_u \) and \( m_v \) are the calculated functional moduli.

Scale spacing ratio \( = m_u / m_v \) was determined with the help of Eq. 1. The limiting values of intensity were determined on the basis of conditions laid down on \( t \) and \( T \).

RESULTS AND DISCUSSION

Mathematical equations

Following the above procedure, the intensity-duration-frequency relationships for 22 stations of Andhra Pradesh were developed and are presented in Table 1. The precision of these equations could be recognized after verifying the reliability of anyone of the station equations. For this purpose, the equation was applied to Khammam station. The maximum per cent deviation between the rainfall intensity values obtained from developed equation \( I = 7.1024T^{0.1629} / (t+0.70)^{1.0531} \) and the observed values obtained from frequency lines from primary data (i.e. probability chart) for various duration and 10, 25 and 50 years frequency ranged from -7.8 to +7.6 per cent (Table 2) while for Andhra Pradesh state as a whole it ranged from -13.7 to +4.4 per cent on the basis of general equation developed for the state \( I = 9.0341T^{0.1828} / (t+0.95)^{1.1131} \) from the 22 stations. This deviation is considered quite low. Not withstanding the inherent weakness of an average equation, the developed equations seems to be quite reliable and may be used
Table 2: Comparison among calculated, nomographic and observed intensities of rainfall (cm hr⁻¹) and their present deviation for Khamman station and the A. P. State

<table>
<thead>
<tr>
<th>Duration Mins: hrs</th>
<th>(i_{\text{cal}}) Frequency (years)</th>
<th>(i_{\text{nomo}}) Frequency (years)</th>
<th>(i_{\text{obs}}) Frequency (years)</th>
<th>(\sigma_i) Frequency (years)</th>
<th>(\sigma_{\text{nomo}}) Frequency (years)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.91 12.66 14.18</td>
<td>11.0 13.0 14.1</td>
<td>11.80 13.70 15.00</td>
<td>-7.5 -7.6 -5.5</td>
<td>-6.8 -5.1 -6.0</td>
</tr>
<tr>
<td>25</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>50</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>15 mins.</td>
<td>8.53 9.90 11.08</td>
<td>8.6 10.0 11.2</td>
<td>8.50 9.60 10.30</td>
<td>0.4 3.1 7.6</td>
<td>1.2 4.2 8.7</td>
</tr>
<tr>
<td>30 mins.</td>
<td>5.90 6.85 7.67</td>
<td>6.0 6.9 7.8</td>
<td>6.40 7.10 7.60</td>
<td>-7.8 -3.5 9.0</td>
<td>-6.3 -2.8 2.6</td>
</tr>
<tr>
<td>1 hr.</td>
<td>2.60 3.02 3.38</td>
<td>2.6 3.0 3.3</td>
<td>2.80 3.20 3.50</td>
<td>-7.1 -5.6 -3.4</td>
<td>-7.1 -6.3 -5.7</td>
</tr>
<tr>
<td>3 hrs.</td>
<td>1.39 1.61 1.81</td>
<td>1.4 1.7 1.8</td>
<td>1.42 1.64 1.80</td>
<td>-2.1 -1.8 0.6</td>
<td>-1.4 3.7 0.0</td>
</tr>
<tr>
<td>6 hrs.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Station: Khamman

General: A. P. State

- \(i_{\text{cal}}\): Calculated intensity of rainfall (cm hr⁻¹) from developed equation;
- \(i_{\text{nomo}}\): Observed intensity of rainfall (cm hr⁻¹) from nomographs of the particular station;
- \(i_{\text{obs}}\): Observed intensity of rainfall (cm hr⁻¹) from the frequency lines from primary data; and
- \(\sigma_i\): Per cent deviation of observed values from the frequency lines to those calculated with the developed equation.
- \(\sigma_{\text{nomo}}\): Per cent deviation of nomographic values from those calculated with the developed equation.
Fig. 3: Nomograph for solving intensity-duration-return period (or recurrence interval) equation-Khiammam.

Fig. 4: Nomograph for solving intensity-duration-return period (or recurrence interval) equation for Andhra Pradesh State.
with confidence.

Nomographs

On the basis of intensity-duration-frequency relationships developed for 22 stations located in Andhra Pradesh (Table 1), nomographs were prepared for all these stations. A nomograph of Khammam station is shown in (Fig. 3) From the nomographs, the rainfall intensity for any desired duration between 10 to 100 year frequency (or return period) could be directly read for a particular location. General nomograph for the state of Andhra Pradesh (Fig. 4) was also developed which may be used in determining intensity for any duration and recurrence interval for any location falling in the state.

Comparison of mathematical and nomographic solutions

Per cent deviation of rainfall intensity observed from nomograph and those calculated from corresponding mathematical equation of Khammam and the state of Andhra Pradesh for various duration and 10, 25 and 50 years frequencies showed that maximum deviation between nomographic solutions and mathematical equations (i.e. $\sigma_{nomograph}$) ranges from -7.1 to +8.7 and -14.4 to +4.2 per cent respectively. The deviations for other 21 stations were still less.

On comparing the rainfall intensity at various duration and frequencies obtained from the developed equations and observed values obtained from the probability charts, it is noted that the maximum deviation ranges from -22.0 to +8.3 per cent for Hanamkonda. Thus, in general, the variations lie around the acceptable limit (+20%)

Limitations of the state equation

When the frequency rainfall intensity of 15, 30 minutes, 1 hour and 3 hours duration for 10, 25 and 50 years for individual station were obtained by using the state equation and compared with computations for individual stations, the per cent deviations were within ±25 per cent (±20 per cent for 15 stations) except for Bapata, Cuddapah, Gannavaram and Ongole where the deviation was beyond ±35 percent. This indicates the limitation of the state equation. The state equation seems to be best suited for location where rainfall is received with intermediate intensity.

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REFERENCES


Fig. A: Rainfall intensities for selected durations and return periods - Khammam

Fig. B: Fitting of constants $b$ and $d$ in the Eq. 1 $= 7.1024/(t+b)^d$ - Khammam
APPENDIX 1

**Step 1:** On log-log paper the values of rainfall intensity for each individual duration were plotted on the Y-axis and return period (or recurrence interval) in years on X-axis (Fig. A). Points were connected for each duration by dotted line giving more weight to the points from 10 year to 100 year return periods and extend the dotted line to cut out the Y-axis against 1 year return period.

**Step 2:** The intensity values, I at different values of t equal to 2.5, 10, 25, 50 and 100 years recurrence intervals for each duration were read from Fig. A. The mathematical relationship between T and various values of I is given by

\[ \log I = m \log T + C \]  

where, \( I \) = maximum intensity for duration \( t \), \( T \) = recurrence interval, \( m \) = frequency factor for each line (i.e., slope of the frequency line) and \( C \) = intercept on Y-axis at \( T = 1 \). These interval equations define the intensity-frequency relationship for any selected duration.

**Step 3:** Then slope \((m)\) of Eq. (a) for each duration was determined and then thier geometric mean \((m)\) was computed. The slope \((m)\) represents the exponent 'a' in equation (1). The geometric mean slope \( m \) thus determined represents actually \( T^a \) in equation (1).

**Step 4:** A line representing geometric mean slope \((m)\) was drawn (Fig. A) at the base through origin; solid lines parallel to this mean slope were drawn to have the lines as close as possible to points between 10 to 100 years return periods extending them to cut the Y-axis. Rainfall intensities against one-year return period for all selected duration were then read on Y-axis.

**Step 5:** Intensity for one-year recurrence interval were plotted on the Y-axis with selected duration \((t)\) on the X-axis on loglog paper (Fig.B) Since the points so plotted did not fall on a straight line, a suitable constant was added \((b)\) to time 't'. Thus, the equation becomes:

\[ I = K/(t+b)^d \]  

------(b)

It is done by trial and error method in such a way that the deviations were minimum (Fig. B).

**Step 6:** The Eq. b written in its logarithmic form is:

\[ \log I = \log K - d \log (t+b) \]  

or \[ \log I - \log K + d \log (t+b) = 0 \]  

...(d)

The constants \( K \) and \( d \) in Eq. d were then solved by the method of least squares. They may be obtained by solving the Eq. e and f.

\[ \log K = \frac{\Sigma \log I \Sigma \log(t+b)^2 - \Sigma \log I \log(t+b) \Sigma \log(t+b)}{N \Sigma \log(t+b)^2 - (\Sigma \log(t+b))^2} \]  

...(e)

and \[ d = \frac{\Sigma \log I \Sigma (t+b) - N \Sigma [\log I \log(t+b)]}{N \Sigma [\log(t+b)^2] - (\Sigma \log(t+b))^2} \]  

...(f)

Thus all the parameters 'a' (step 3), 'b' (step 5) and 'k' & 'd' (step 6) become known for equation (1).

**Step 7:** At this stage, frequency factor \( T \) obtained in step 3 above was included to give finally the intensity-duration-frequency or return period formula

\[ I = \frac{K}{(t+b)^d} T^a \]