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Soil temperature and its dependence on soil properties

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ABSTRACT

A scheme is developed to compute the thermal properties of the soil, by using the 24-hour temperature values at a depth of 5 cm., at intervals of 1 hour. The computed thermal properties, such as, thermal conductivity, thermal diffusivity, etc., are used to predict the surface temperatures over a period of 24 hours, starting from an initial value. The prediction equation is very simple and consists only of a time tendency term for the surface temperature and a trigonometric forcing term, which is a function of the damping depth.

Key words: Soil temperature, Thermal properties of soil, Prediction

It is known that solar radiation is received by the earth's surface and transferred to the atmosphere in the form of sensible heat, latent heat and long-wave radiation. At the ground surface the soil temperature results from the overall balance between the net all-wave radiation, (R_N) received at the surface, the sensible heat flux, (H_0) directly resulting in temperature change in the atmosphere, the ground heat flux, (G_0) which affects the temperature change to the ground surface and the latent heat flux, (LE_0) which occurs due to the evaporation of water-vapour at the surface. Thus, the energy balance equation at the surface is as follows:

$$R_N = H_0 + LE_0 + G_0$$

An accurate and simple method to determine the thermal properties of the soil would lead to an accurate evaluation of G_0. The surface temperature is defined as the mean temperature of a thin slab of the soil of thickness 1 cm. The soil temperature measured at a depth of 5 cm would not have problems associated with exposure of the temperature sensor to the direct radiation and also the damping of the surface temperature wave would be small. In practice, the thermal properties of the soil are usually computed from the temperature range (difference between the maximum and minimum temperatures), at a particular depth. However, it is difficult to identify the exact maximum and minimum temperatures. In this study, an expression is derived to compute the damping depth in terms of the integrated temperature differences between two time intervals during a 24-hour period. These temperature differences are obtained from a Fourier representation of the soil temperature during the 24-hour period, and are free from subjective errors.

In this study, the following problems are attempted to be addressed

(a) Develop a method of evaluating the thermal properties of the soil such as thermal conductivity (k_S) thermal diffusivity (K_S) etc. from the 24-hour temperature data at a depth of 5 cm.

(b) Predict the surface temperature values over a period of 24-hours, starting from an initial value and using the mean values of the thermal properties of the soil evaluated in (a).

MATERIALS AND METHODS

Physics of the model

It is assumed that the slab under consideration is either infinitely thin or has a temperature θ_g, which is independent of depth, so that the rate of change of temperature of the slab is dictated by the imbalance of fluxes between a forcing term (H) and a restoring term (Y). Thus, the predictive equation for surface soil temperature can be written as

$$C_g \frac{\partial \theta_g}{\partial t} = H - Y \quad \text{... (1)}$$

where, ($C_g = C_s \Delta Z$) C_s being the volumetric heat capacity of the soil and ΔZ is the thickness of the slab. C_g is called the thermal inertia. The forcing term (H) is the ground heat flux (G_0) obtained from the energy balance equation.

$$H = G_0 = R_N - H_0 - E_0 \quad \text{... (2)}$$

The restoring term (Y) is given by

$$Y = -\mu (\theta_g - \overline{\theta}) \quad \text{... (3)}$$

where 'μ' is the coefficient of heat transfer and $\overline{\theta}$ is the mean substrata constant temperature.

We consider a thin layer of soil of thickness, ΔZ. The time rate of change in the temperature of the soil layer, neglecting the horizontal conduction of heat in the soil, is given by

$$\rho c \ \Delta Z \frac{\partial \theta_g}{\partial t} = \frac{\partial G}{\partial Z} \Delta Z \quad \text{... (4)}$$

where ρ is the density of the soil and c is its specific heat (J $kg^{-1} K^{-1}$). The R.H.S. of Eq.(4) represents the difference in the heat fluxes into and out of the layer. If the soil layer is very close to the surface, then the R.H.S. can be expressed as :

$$\frac{\partial G}{\partial Z} = G_0 - G_1$$

where G_1 is the heat flux at depth ΔZ from the surface. Eq. (4) now becomes

$$C_s \frac{\partial \theta_g}{\partial t} = G_0 - G_1 \quad (2.5)$$

where $C_s = \rho c$, is the volumetric heat capacity of the soil ($Jm^{-3}K^{-1}$).

The subsurface heat flux at any depth Z can be evaluated from Fourier's law for heat conduction in a homogeneous body, as

$$G = -k_s (\partial \theta_g / \partial Z) \quad \text{... (6)}$$

Where k_s is the thermal conductivity of the soil ($Wm^{-1} K^{-1}$).

$k_s = C_s K_s$,

where K_s is the thermal diffusivity ($m^2 s^{-1}$) Substituting in Eq.(4), we get

$$\frac{\partial \theta_g}{\partial t} = K_s \frac{\partial^2 \rho \, \theta_g}{\partial Z^2} \quad \text{... (7)}$$

In the case of diurnal forcing, we take the upper boundary condition at Z=0, as

$$\theta_g(0, t) = \theta_0 (t) A_0 \operatorname{Sin} (\Omega t)$$

where $\Omega = 2\pi/P$, 'P' being the time period of the diurnal temperature wave, and θ_0 is the surface temperature at Z=0. (2 A_0) is the difference between the maximum and minimum temperature during the day. For the lower boundary we have

$\theta_g(\infty, t) = \overline{\theta}$,

the deep soil temperature.

Combining the two boundary conditions, the solution of Eq. (7) is given by

$$\theta_g(Z, t) = A_0 \exp(-C_1 Z) \sin(\Omega t - C_1 Z) + \overline{\theta} \quad ...(8)$$

We define a damping depth (d), as the depth of soil across which the temperature difference is (1/e) times the diurnal amplitude. The constant C_1 is identified as the reciprocal of the damping depth, i.e. $C_1 = 1/d$. Substituting Eq. (8) in Eq. (7) we get

$(C_1)^2 = \Omega / (2 K_s)$, or $d = \sqrt{(2K_s/ \Omega)}$

$$\theta_g(Z,t) = A_0 \exp(-Z/d) \sin(\Omega t - Z/d) + \overline{\theta} \quad ...(9)$$

The soil heat flux at a depth Z can be written from Eq. (6) as

$$G(Z,t) = -(\Omega C_S k_s/2)^{1/2} [\theta_g(Z,t) - \overline{\theta} + 1/\Omega \, (\partial \theta_g / \partial t)] \quad ...(10)$$

$G_0 = G(0,t)$

and

$\theta_g(0,t) = (1/\delta) \int \theta_g (Z,t)\, dZ$,

when ΔZ is very small and equal to δ Substituting in Eq.(10), we get

$$\alpha \frac{\partial \theta_s}{\partial t} = \frac{2}{C_s d} G_0 - \Omega (\theta_g - \overline{\theta}) \quad ...(11)$$

Here $\alpha = (1 + 2\delta/d)$. This is the force-restore formulation.

Method of computation

Differentiating Eq. (9) with respect to time, we get

$$\frac{\partial \theta_s}{\partial t} = A_0 \exp(-Z/d)\, \Omega \cos(\Omega t - Z/d) \quad ...(12)$$

Now integrating the above equation with respect to time, between the time limits t_1 and t_2, we get the total change in the surface temperature as

$$\nabla\theta_1 = \int_{t}^{t_2} (\partial\theta_g / \partial t)\, dt$$

Substituting for $(\partial\theta_g / \partial t)$ from Eq. (12), we get the following expression

$$\nabla\theta_1 = 2 A_0 \exp(-Z/d) \cos(Z/d) \sin\{\Omega (t_2 - t_1)/2\}[\cos\{\Omega (t_2 + t_1)/2\} + \sin\{\Omega (t_2 + t_1)/2\}\tan(Z/d)]$$

In the same way, the total change in the surface temperature between the limits t_2 and t_3 is given by

$$\nabla\theta_1 = 2 A_0 \exp(-Z/d) \cos(Z/d) \sin\{\Omega (t_3 - t_2)/2\}[\cos\{\Omega (t_2 + t_3)/2\} + \sin\{\Omega (t_2 + t_3)/2\}\tan(Z/d)]$$

Dividing $\nabla\theta_1$ by $\nabla\theta_2$, we get the following expression

$$\frac{\nabla\theta_2 \sin\{\Omega (t_2 - t_1)/2\}[\cos\{\Omega (t_2 + t_1)/2\} - \nabla\theta_1 \sin\{\Omega (t_3 - t_2)/2\}\cos\{\Omega (t_2 + t_3)/2\}}{\nabla\theta_1 \sin\{\Omega (t_3 - t_2)/2\}\sin\{\Omega (t_2 + t_3)/2\} - \nabla\theta_2 \sin\{\Omega (t_2 - t_1)/2\}\sin\{\Omega (t_2 + t_1)/2\}} = \tan(Z/d) \quad ...(13)$$

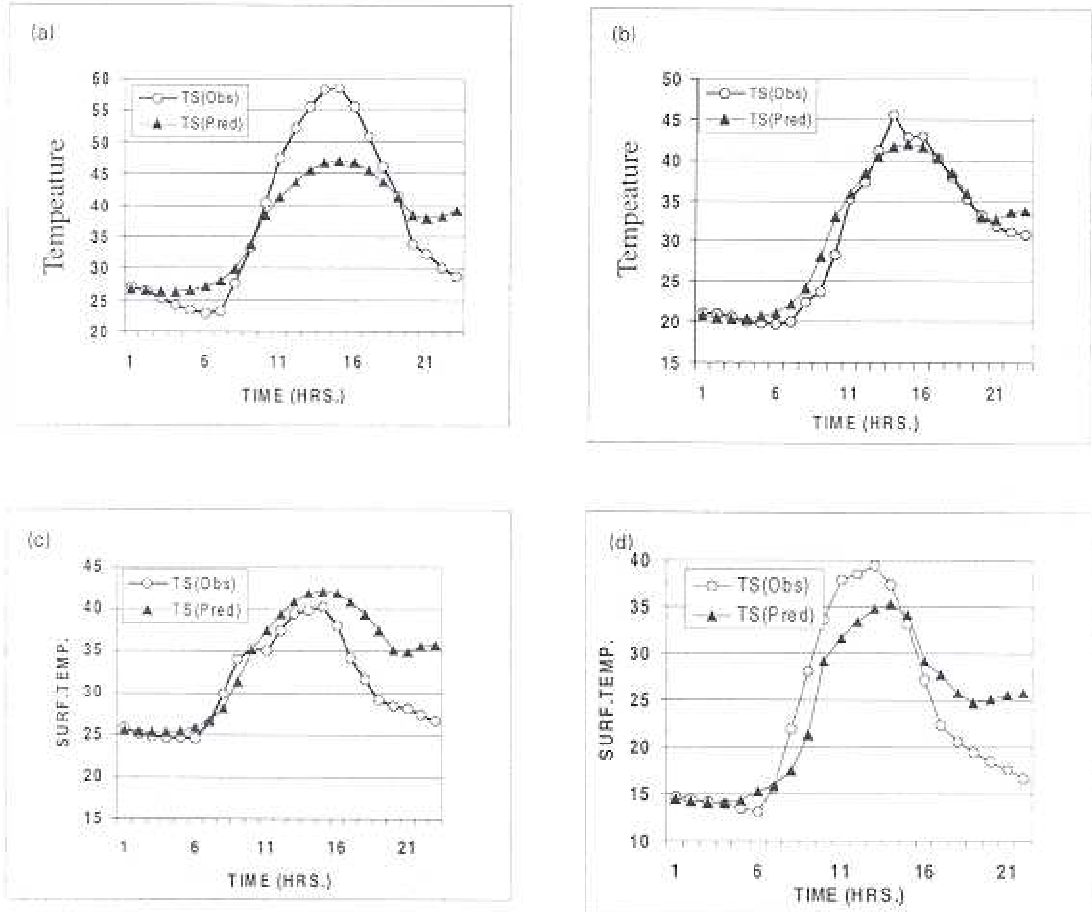

Fig.1: Predicted and observed surface temperatures at Anand, over a 24-hour period, (a) 15.5.97 (b) 15.7.97 (c) 15.9.97 (d) 15.12.97

The value of the damping depth, 'd', can be obtained from this expression, provided the values of $\nabla\theta_1$ and $\nabla\theta_2$ are known. A Fourier analysis is done for the 24-hour temperature values at the depth of 5 cms. The reconstructed temperature field is given by

$$(\theta_F)_k = A_k \, \mathrm{Cos}\{(2\pi/\rho)kt\} + B_k \, \mathrm{Sin}\{(2\pi/\rho)kt\} + \theta + \theta_N \quad \ldots (14)$$

Here, k represents the wave number and A_k, B_k are the Fourier coefficients. From (14) the values of $\nabla\theta_1$ and $\nabla\theta_2$ can be obtained. The thermal diffusivity can be obtained from the damping depth as follows

$$K_S = d^2 \, \Omega/2$$

Substituting the respective trigonometric functions for the terms in Eq. (10), we get the

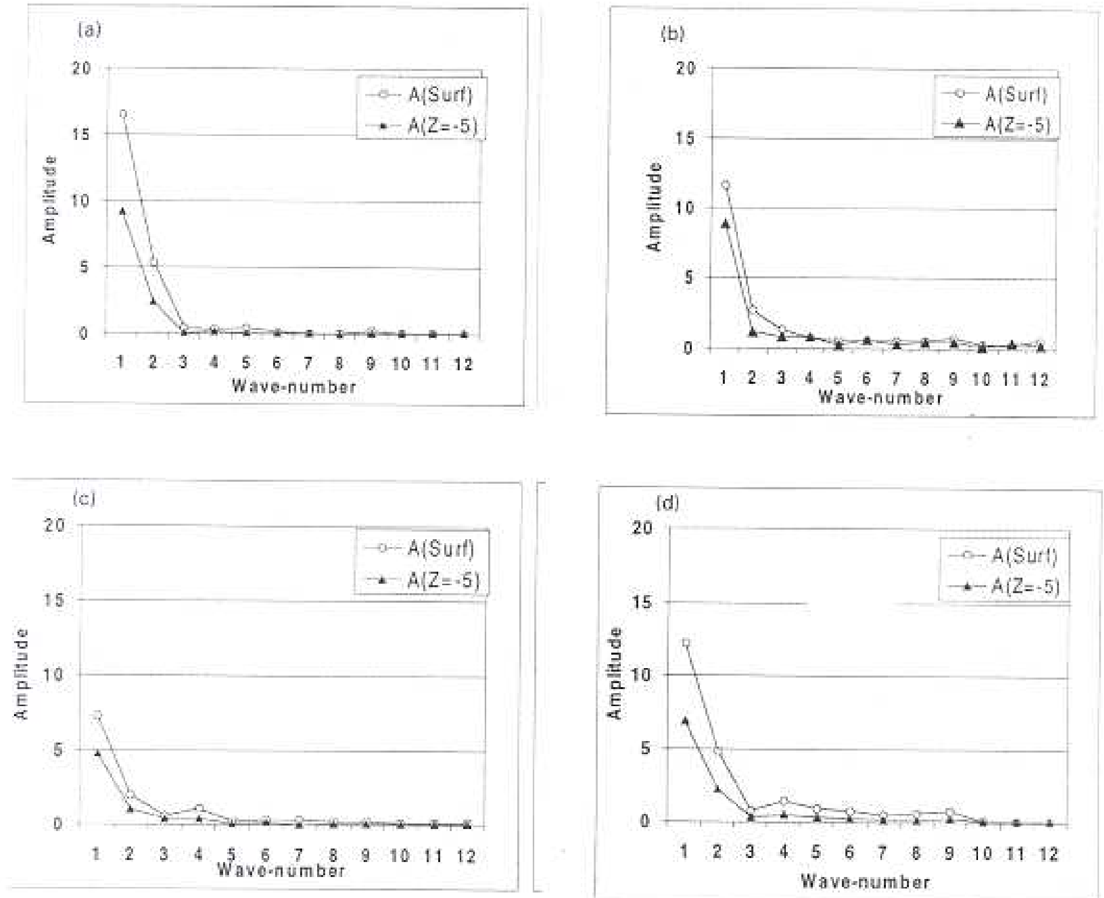

Fig.2 : Amplitudes of the various wave components of the temperature wave at the surface, on (a) 15.5.97 (b) 15.7.97 (c) 15.9.97 (d) 15.12.97

ground heat flux as

$$G(Z,t) = -A_0 \exp(-Z/d) \sqrt{2} (k_s / d) \, Cos\{\pi/4 - (\Omega t - Z/d + \varnothing)\} \quad ...(15)$$

If the ground heat flux is measured at a depth of 5 cms., the above equation gives the value of (k_s / d), from which the thermal conductivity of the soil can be obtained.

Prediction of the surface temperature

If the surface temperature at the initial time, t_o, (zero hours in this case) is known, then the temperature at any time t_n, can be computed from the equation.

$$(\theta_g)_n = (\theta_g)_o + \nabla\theta_n$$

where

$$\nabla\theta_n = \int_{t_0}^{t_n} (\partial\theta_g / \partial t)\, dt \qquad \ldots (16)$$

Substituting the trigonometric expressions for each term on the R.H.S. of Eq. (11), we have the expression for the time integral as follows:

$$\int_{t_0}^{t_n} (\partial\theta_g / \partial t)\, dt = 2\, A_0 / \alpha\, \mathrm{Sin}\{\Omega(t_n + t_0)/2 + \varnothing\}\ \mathrm{Sin}\{\Omega(t_n + t_0)/2\}(N2\text{-}1) \qquad \ldots(17)$$

to

Here α is expressed as follows :

for $t < 10$

$\alpha = (1+2\delta / d).[1- \mathrm{Sin}\, (\Omega t + \varnothing)]$;

for $10 \leq t \leq 20$

$\alpha = 0.4\, (1+2\delta/d)$; and

for $t > 20$

$\alpha = 0.6\, (1+2\delta/d).[1+ \mathrm{Sin}\, (\Omega t + \varnothing)]$

RESULTS AND DISCUSSION

Table 1 shows the mean values of the thermal properties of the soil at the experimental site at Anand, on the dates shown. It may be seen that in the month of September, the volumetric heat capacity of the soil is very high and correspondingly, the damping depth and thermal conductivity are small. The gravimetric soil moisture content, which was evaluated on 16.9.97, revealed a fairly high value of 13.71%. It is possible that the computed high value of the volumetric heat capacity of the soil is due to the high moisture content.

Fig. 1 shows the observed and the predicted surface temperatures during the 24-hour period, on different dates. The predicted temperatures show good agreement with the observed surface temperatures up to around 18 hours, in July and up to around 14 hours in September. Beyond this time range, the predicted temperatures begin to diverge from the observed temperatures. The moisture content of the soil is the highest during these months, resulting in smaller damping depths. It can be seen from Eq.12 that the time tendency term for surface temperature is small if the damping depth is small, causing smaller errors in the evaluation of the integral in (Eq.16). Meteorological observations indicate cloud cover of 5 oktas in July and September, resulting in smaller amplitude of the surface temperature. This partly explains the better agreement between observed and predicted values in July and September.

In the months of May and December, the computed amplitudes of the surface temperature wave are large, resulting in larger values of the surface temperature tendency

TABLE 1 : Thermal Properties of the soil at Anand.

Date	$k_S(Wm^{-1})$	$K_S(m^2s^{-1})x10^{-6}$	$C_S(Jm^{-3}K^{-1})x10^6$	$(k_S/d)(Wm^{-2})$
15.5.97	1.006 ± 0.23	0.764	1.32	6.94
15.7.97	0.52 ± 0.22	0.425	1.22	4.8
15.9.97	0.31 ± 0.15	0.091	3.39	6.17
15.12.97	0.727 ± 0.15	0.552	1.32	5.9

term. Since this prediction method of surface temperature does not consider the soil heat flux explicitly, the error in the computation of the temperature tendency term is large, owing to the absence of G_0. The divergence of the predicted temperature from the observed values in the evenings, during all the cases considered, may also be attributed to this cause, mainly due to the reversal of the soil heat flux. It may be seen from Eq (17) that the temperature increment $\nabla\theta_n$, depends only on the estimated temperature amplitude A_0, at the surface, which is estimated from the temperature amplitude at the depth of 5 cms by the factor exp(Z/d). Fig. 2 shows the amplitudes of the different wave components, obtained through a harmonic analysis of the surface and 5 cm. depth temperatures, over a 24 - hour period, on the given dates. It can be seen that for wave number one, the difference between the amplitudes at the surface and at the 5 cm depth is large in May and December compared to that in July and September, resulting in larger errors in the time tendency term for surface temperature.

It is seen that even a very simple scheme can predict the soil temperatures fairly well for a period of up to six hours, if the values of the thermal properties of the soil are determined accurately. For better results, the force-restore formulation embodied in (Eq.11) should be used, because it contains the soil heat flux term and the volumetric heat capacity, which is a function of the soil moisture.

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REFERENCES

Dickinson, R. E. 1988. The force - restore model for surface temperature and its generalizations. *J. Climate*, 1 : 1086-1097

Lyons, T. and Scott, B. 1990. Principles of Air Pollution. Belhaven Press. Pinter Publishers, London. pp. 219.

Smith, E. A., Crosson, W. L. and Tanner, B. D. 1992. Estimation of surface heat and moisture fluxes over a prairie grassland. *J. Geophys. Res.*, 97 : 18557-18582.