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Empirical relationship of sixth order to estimate global solar radiation from hours of sunshine

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ABSTRACT

A new set of constants based on sixth order polynomials fitting of measured data of sixteen Indian cities has been found to estimate the monthly mean global radiation on a horizontal surface. These constants provide good estimates of monthly mean global radiation on a horizontal surface with a maximum deviation of 8%. A comparison of the present result with the other models shows that the new constants yield more accurate results.

Key words: Monthly mean global solar radiation, Sunshine hours, Percentage estimation

A reasonably accurate knowledge of the amount of the global irradiation at any place is necessary for many solar energy applications. While solar energy data are recognized as very important, their acquisition is by no means a straight forward. The necessary equipments such as pyranometer and pyrheliometers for their measurements are available only at a few places. Consequently, adequate facilities are often not available in developing countries to mount viable monitoring programmes. For this reason, their have been attempts from theoretical models (Angstrom 1924; Singh *et al.* 1996; Elagib *et al*. 1999, Elagib *et al*. 2000 and Togrul *et al.* 2000).

There are various empirical correlations to estimate total solar radiation incident on a horizontal surface. The first empirical correlations proposed by Angstrom (1924) and modified by Prescott (1940) and Page (1961) are being used most correctly and widely to estimate global irradiance from bright hours of sunshine. Using bright sunshine and global radiation data of 48 locations around the world, a correlation of third order was developed by Bahel *et al*. (1987). Several investigators (Elagib *et al*. 1999) have found that non-linear relationship provide slightly better result.

The objective of the present study is to derive new set of constants for empirical relationship of sixth order to estimate monthly mean daily values of global radiation from sunshine hours.

THEORY

The first empirical correlations proposed by Angstrom (1924) and modified by Prescott (1940) and Page (1961), which correlates global solar radiation and bright hours of sunshine.

$$\frac{\overline{H}}{\overline{H}_o} = a_1 + b_1\left(\frac{\overline{S}}{\overline{S}_O}\right) \tag{1}$$

where a_1, and b_1 are constants, $\overline{H}$ is the monthly mean global radiation on horizontal surface, $\overline{S}$ is the measured monthly mean bright sunshine hours, $\overline{S}_o$ is maximum possible monthly mean sunshine hours and $\overline{H}_o$ is monthly mean daily extraterrestrial radiation and daily values of extraterrestrial radiation may be obtained by the following equation (Klein 1977).

$$H_o = \frac{24}{\pi} I_{sc}\left[1 + 0.33\cos\left(\frac{360 D_n}{365}\right)\right]\left[\cos L \cos\delta \sin\omega_s + \frac{2\pi\omega_s}{360}\sin L \sin\delta\right] \tag{2}$$

Where sunset hour angle

$$\omega_s = \cos^{-1}(-\tan L \tan \delta) \tag{3}$$

I_{sc} is the solar constant. L is the latitude of location under consideration; D_n is day of year starting from first January and δ is declination as given below.

$$\delta = 23.45 \sin\left[\frac{360(284 + D_n)}{365}\right] \tag{4}$$

S_o, maximum possible sunshine hours and can be calculated from Cooper's formula 1969.

Table 1: Latitude and Longitude of different cities under consideration.

Sr. No.	Stations	Latitude (Degrees)	Longitude (Degrees)	H Meters	Location
1	Trivandrum	8.48	76.95	0060	Coastal
2	Kodaikanal	10.23	77.47	2329	Coastal
3	Madras	13.00	80.18	0010	Coastal
4	Goa/Panji	15.48	73.82	0058	Coastal
5	Vishakhapatnam	17.72	83.23	0003	Coastal
6	Poona	18.53	73.85	0555	Coastal
7	Bombay	19.12	72.85	0008	Coastal
8	Nagpur	21.10	79.05	0308	Inland
9	Bhaunagar	21.75	72.20	0005	Inland
10	Ahmadabad	23.07	72.63	0055	Inland
11	Calcutta/Dum Dum	22.65	88.45	0004	Coastal
12	Shilong	25.57	91.88	1598	Inland
13	Jodhpur	26.30	73.02	0217	Inland
14	Kanpur	26.47	80.40	0127	Inland
15	Lucknow	26.76	80.88	0128	Inland
16	New Delhi	28.58	77.20	0211	Inland

$$S_o = \frac{2}{15}\cos^{-1}((-\tan L \tan\delta)) \tag{5}$$

Different values of regression coefficients have been proposed by Reitveld 1978 for different locations across the globe.

Using bright sunshine and global radiation data of 48 locations around the world a correlation of third order was developed by Bahel *et al.* 1987. Several investigators have found that nonlinear relationship of the following type provide better result.

$$\frac{\overline{H}}{\overline{H}_o} = a_2 + b_2\left(\frac{\overline{S}}{\overline{S}_o}\right) + c_2\left(\frac{\overline{S}}{\overline{S}_o}\right)^2 \tag{6}$$

where a_2, b_2 and c_2 are constants.

Power, exponential and logarithmic correlations have also been tested by several investigators [Elagib *et al*. 2000, Togrul *et al.* 2000].

$$\frac{\overline{H}}{\overline{H}_o} = [a_3 \exp\left(b_3 \frac{\overline{S}}{\overline{S}_o}\right)] \tag{7}$$

$$\frac{\overline{H}}{\overline{H}_o} = [a_4\left(\frac{\overline{S}}{\overline{S}}\right)^{b_4}] \tag{8}$$

$$\frac{\overline{H}}{\overline{H}_o} = a_5 \log\left(\frac{\overline{S}}{\overline{S}_o}\right) + b_5 \tag{9}$$

where a_3, b_3, a_4, b_4, a_5 and b_5 are constants.

Besides, all above empirical correlations discussed, a sixth order polynomial could also be fitted:

$$\frac{\overline{H}}{\overline{H}_o} = a_6 + b_6\left(\frac{\overline{S}}{\overline{S}_o}\right) + c_6\left(\frac{\overline{S}}{\overline{S}_o}\right)^2 + d_6\left(\frac{\overline{S}}{\overline{S}_o}\right)^3 + e_6\left(\frac{\overline{S}}{\overline{S}_o}\right)^4 + f_6\left(\frac{\overline{S}}{\overline{S}_o}\right)^5 + g_6\left(\frac{\overline{S}}{\overline{S}_o}\right)^6 \tag{10}$$

where a_6, b_6, c_6, d_6, e_6, f_6, and g_6 are constants.

DATA AND METHODOLOGY

Analysis has been carried out for sixteen Indian locations. Measured data of global solar radiation on a horizontal surface and bright sunshine hours have been downloaded from the wave site of World Radiation Data Centre for the years 1991, 1992, 1993 except for the cities of Kanpur and Lucknow, which we measured using a precision pyranometer. A least square regression analysis has been carried out using standard software, which is available on the internet [http:// www3.sympatico.ca] Standard deviation and correlation coefficients have also been calculated. Since we do not want to rank different models,

Table 2: Regression coefficients of equations (1) and (6) for different cities of India

City	equation (1)		
	a_1	b_1	a_2
Trivandrum	0.289	0.509	0.256
Kodaikanal	0.276	0.620	0.188
Madras	0.297	0.444	0.470
Goa/Panji	0.259	0.536	0.222
Vishakhapatnam	0.349	0.363	0.211
Poona	0.246	0.472	0.292
Bombay	0.312	0.364	0.274
Nagpur	0.219	0.519	0.225
Bhatnagar	0.326	0.428	0.170
Ahmadabad	0.309	0.393	0.216
Calcutta	0.291	0.390	0.299
Shilong	0.049	0.559	0.228
Jodhpur	0.413	0.297	0.725
Kanpur	0.338	0.370	0.769
Lucknow	0.309	0.364	0.062
New Delhi	0.213	0.588	-0.326

we selected percentage estimation as a single parameter to show superiority of our model over different other models. Percentage estimation is defined as

$$\frac{Estimated\ \ value\ \ of\ \ radiation}{Measured\ value\ of\ \ radiation}\times 100$$

An ideal empirical correlation should give 100% estimation for each month of the year.

RESULTS AND DISCUSSIONS

Latitude and longitude of different locations under consideration are given in Table 1. Regression coefficients, which have been determined using equations (1), (6) – (10) are listed in Tables 2,3 and 4 for different locations of India.

Data of $\bar{H}/\bar{H}_0$ for Lucknow, Nagpur and Trivandrum (Fig. 1) shows least scattering from well-known Angstrom – Prescott Equation. Performances of equations (1) and (6) to (9) for such locations are almost same (percentage estimation ± 6.0 from 100%). Equation (10) gives slightly better result with percentage of estimation ± 3.0 from 100% (Fig. 2). We started with second order polynomial gone up to tenth order polynomial for each location under study and find that only sixth order polynomial provides better result. For other locations scattering is obvious (Fig. 3). For these locations estimated values of monthly mean daily values of global solar radiation on a horizontal surface varies significantly as determined from equations (1) and (6) to (9). For such locations, equation (10) provides better result (Fig. 4) (percentage of estimation ±5.0 from 100%). For Kanpur, we observe worst scattering in data of $\bar{H}/\bar{H}_0$. Percentage of estimation varies from -13 to 24 for equation (1), -12 to 20 for equation (6) , -14 to 24 for equation (7), -14 to 23 for equation (8) and -15 to 17 for equation (9). For sixth order polynomial (equation (10)), it varies only ±8.0 from 100%.

Regression coefficients a_1, b_1, a_3, b_3, a_4, b_4, a_5 and b_5 varies from 0.049 to 0.349, 0.297 to 0.620 , 0.644 to 0.849, 0.319 to 0.685 , 0.268 to 0.368 , 0.326 to 1.158 , 0.213 to 0.748 and 0.177 to 0.710 respectively. For New Delhi, two coefficients (a_2 and c_2) are negative and for Kanpur, Madras and Jodhpur only the coefficient b_2 is negative. For other cities and in general coefficient c_2 is only negative. Values of regression coefficients determined for equation (10) for the cities like Jodhpur, Kanpur, Lucknow and New Delhi is abruptly much higher than other cities under consideration. These cities are having almost same kind of climatic conditions and sea has no effect on their climate. Other cities except Poona and Shilong are the coastal cities and having comparable values of regression coefficients. Regression coefficients of sixth order polynomial are highly site-dependent and could not be averaged out.

CONCLUSIONS

Unlike other models, the percentage of estimation of sixth order polynomial fitting is only ± 8 from 100 percent for the entire year at different locations of India. Sixth order polynomial provides better result than any other empirical equation suggested in literature using sunshine hours only. However, regression coefficients of sixth order polynomial

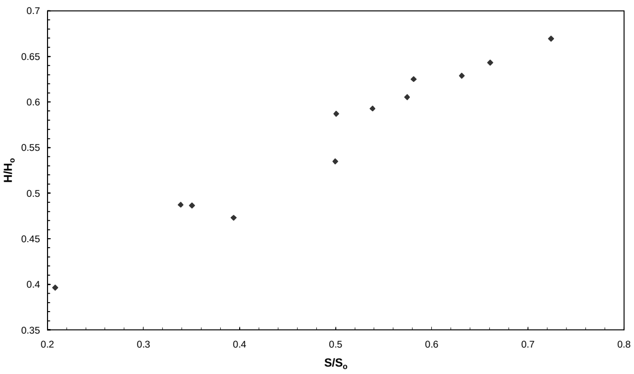

Fig. 1: Variation of S/S$_o$ against H/H$_o$ in Trivandrum

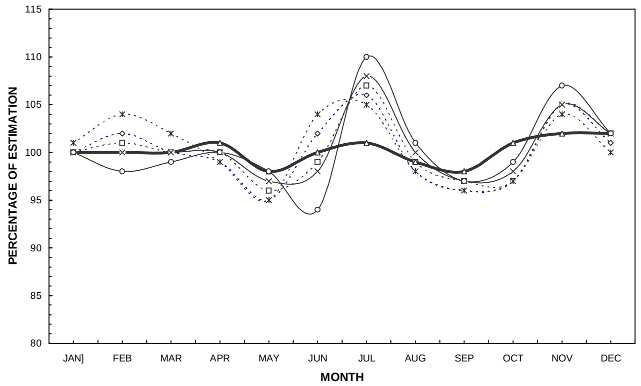

Fig. 2: Percentage of estimation of monthly mean global radiation for Trivandrum. (--◇--), (--□--),(−−X−−), (--*--), (−−○−−), (—△—) represents equations (1), (6)-(9), and (10) respectively.

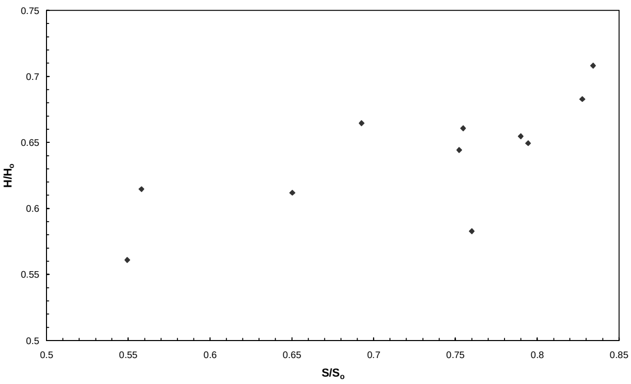

Fig. 3: Variation of S/S$_o$ against H/H$_o$ in Jodhpur

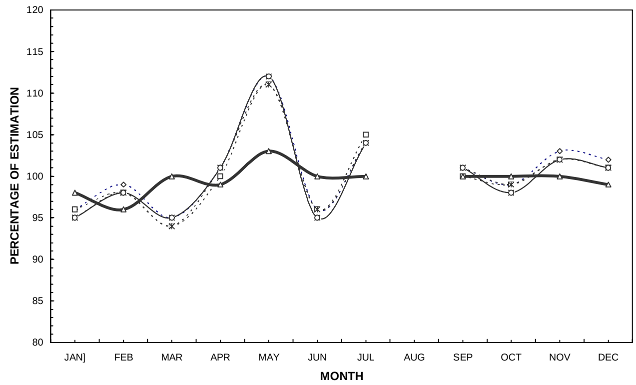

Fig. 4: Percentage of estimation of monthly mean global radiation for Jodhpur. (--◇--), (--□--), (−−X−−), (--*--), (−−○−−), (—△—) represents equations (1), (6)-(9), and (10) respectively

Table 3 : Regression coefficients of equations (7) (8) and (9) for different cities of India

City	equation 7		equation 8		equation 9	
	a_3	b_3	a_4	b_4	a_5	b_5
Trivandrum	0.767	0.436	0.360	1.021	0.731	0.228
Kodaikanal	0.849	0.503	0.328	1.105	0.661	0.280
Madras	0.719	0.466	0.352	0.769	0.710	0.268
Goa/Panji	0.758	0.468	0.298	1.051	0.731	0.232
Vishakhapatnam	0.687	0.344	0.367	0.702	0.671	0.177
Poona	0.672	0.356	0.325	0.326	0.682	0.177
Bombay	0.647	0.319	0.325	0.778	0.632	0.145
Nagpur	0.703	0.506	0.268	1.093	0.677	0.238
Bhaunagar	0.735	0.415	0.350	0.812	0.715	0.217
Ahmadabad	0.683	0.393	0.334	0.783	0.668	0.195
Calcutta	0.644	0.396	0.321	0.810	0.624	0.189
Shilong	0.729	0.429	0.282	1.191	0.689	0.197
Jodhpur	0.713	0.338	0.444	0.502	0.213	0.710
Kanpur	0.657	0.329	0.368	0.665	0.656	0.189
Lucknow	0.668	0.459	0.359	0.643	0.661	0.261
New Delhi	0.800	0.685	0.278	1.158	0.748	0.396

Table 4: Regression coefficients of 6th order polynomial (equation (10)) for different cities of India.

City	a_6	b_6	.c_6	d_6	e_6	f_6	g_6
Trivandrum	-14.2	202.1	-1103.2	3075.6	-4645.1	3626.0	-1148.8
Kodaikanal	3.3	-54.9	391.6	-1379.2	2579.4	-2439.1	914.9
Madras	-57.9	607.9	-2594.1	5803.9	-7179.9	4659.6	-1240.3
Goa/Panji	-5.9	99.6	-591.9	1725.5	-2637.4	2031.0	-621.6
Vishakhapatnam	3.2	-31.6	132.4	-260.3	258.1	-121.6	20.6
Poona	7.5	-126.5	830.6	-2587.1	4131.4	-3270.5	1017.9
Bombay	-0.6	17.1	-107.6	336.0	-543.7	437.4	-138.3
Nagpur	8.7	-126.8	744.1	-2175.1	3376.3	-2660.9	838.3
Bhaunagar	4.5	-72.3	460.6	-1403.9	2217.9	-1748.3	542.9
Ahmadabad	-40.7	557.0	-2913.5	7548.4	-10314.9	7127.0	-1963.6
Calcutta	120.2	-1615.5	8855.7	-25284.5	39698.8	-32525.0	10871.3
Shilong	4.41	-93.3	786.7	-3160.2	6534.3	-6692.6	2685.3
Jodhpur	-9548.5	83762.7	-304722.0	588492.5	-636355.4	36533.9	-86994.9
Kanpur	-4403.9	45088.2	-188221.5	411457.7	-497958.8	316921.4	-82990.2
Lucknow	-788.0	6597.75	-22837.6	41857.6	-42847.4	23230.5	-5212.8
New Delhi	-6326.6	66694.4	-290381.6	668728.9	-859459.2	584685.8	-164539.1

are highly site dependent.

REFERENCES

Angstrom, A. (1924). Solar and terrestrial radiation. *Quart. J. Royal Meteorol. Soc.*, **50:**121-126.

Bahel, V., Bakhsh, H., and Srinivasan, R. (1987). A correlation for estimation of global solar radiation. *Energy*, **12**:131-135.

Cooper, P. I. (1969). The absorption of radiation in solar stills. *Solar Energy*, **12**:333-346.

Elagib, N.A., Lava, S.H., and Mansel, M.G. (1999).Correlation between clearness index and relative sunshine duration over Sudan. *Renewable Energy*, **17(4)**: 473-498.

Elagib, N.A., Mansel, M.G. (2000). New approach for estimating global radiation across Sudan. *Energy Conv. Manag.*, **41**: 419-434.

Klein, S. A. (1977). Calculation of monthly average insolation on tilted surface. *Solar Energy*, **19**:325-329.

Page, J.K. (1961). The calculation of monthly means solar radiation for horizontal surface and inclined surface from sunshine records for latitudes 40° N-40° S. *Proc. U.N. Conf. New Sources Energy*, **4:** 378-390.

Prescott, J. A. (1940). Evaporation from water surface in relation to solar radiation Trans. *Roy. Soc. Australia*, **64**:114.

Rietveld, M. R. (1978). A new method to estimate the regression coefficient in the formula relating solar radiation to sunshine. *Agricul. Meteorology*, **19**:243-252.

Singh, O. P., Srivastava, S. K., and Gaur (1996). A., Empirical relation to estimate global radiation from hours of sunshine. *Energy Conv. Manag.*, **37(4)**: 501-504.

Togrul, I.T., Tugrul, H., and Evin, D. (2000). Estimation of global solar radiation under clear sky radiation in Turkey. *Renewable Energy*, **21**: 271-287.

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