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Stochastic modelling of water deficit

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ABSTRACT

A stochastic model for weekly water deficit series, using 28 years climatological data, under climatic condition of S.K. Nagar has been developed. The turning point test and Kendall's rank correlation test are applied for detecting the trend. Correlogram technique is used to detect the periodicity, which is then analyzed by Fourier series method. Significant harmonics were also identified. The statistical properties of the generated water deficit series were compared with observed series. The developed model was validated by predicting two years ahead and compared with the observed water deficit series, The test results indicated the high degree of model fitness. The developed model may be used for representing the time-based structure of the water deficit time series.

Key words: stochastic, water deficit, time series, periodic.

Forecasting of water deficit particularly in water resources project planning, design and operation is of paramount importance. It is increasingly recognized that time series analysis is of considerable practical use in dealing with forecasting of hydrological variables (Salas *et al.,* 1980). Employing a mathematical model that represents the stochastic process of water deficit, the likely synthetic sequences of future water deficit values can be obtained. The analysis of time series is done to understand the mechanism that generate the data and to produce likely future sequences, if required. These are attempted by making inferences regarding the underlying laws of the stochastic process from one or more sequences of recorded observations and then to postulate a model that fits the data, which are again used for estimation purposes. At first it is necessary to identify and analyze the different components of time series and then generate the future sequences (Kottegoda, 1980).

The stochastic modelling of water deficit as time series is important for selection of suitable crop variety, scheduling of irrigation and drought management planning. During the past years many investigators (Box and Jenkins, 1976; Jolliffe, 1983; Mutua, 1998; Jat *et al*., 2003) have analysed the time series of rainfall, stream flow and other climatic parameters for generation of data. However, no such study was made for this region. With this in view the present study was undertaken to develop a suitable model for data generation of water deficit for the region.

MATERIALS AND METHODS

Weekly water deficit of S.K. Nagar (24.32^0 N Lat., 72.32^0 E Long. and 154.52 m alt.) were used for this study. From the water deficit data of thirty years (1974-2003), the data up to 2001 were used for model development and the remaining two years data (2002-03) were used in validating the performance of the model.

Time series model development

The time series of water deficit was decomposed into a deterministic component in the form of trend and periodic parameter and a stochastic (random) component consisting of chance and chance-dependent effects. A general additive form provides a reasonable model in most cases and is expressed as:

$$X_t = T_t + P_t + S_t \qquad \dots (1)$$

Where T_t is the trend component; P_t is the periodic component; and S_t is stochastic component having dependent and independent parts, at time t. The first two components in the model that described the time series are deterministic in nature. Modelling of the water deficit series through decomposing into periodic and stochastic components allows one to develop an improved methodology for generation of weekly water deficit series. Each of the model components is, therefore, analyzed and determined stepwise according to the procedure described in the following subsection.

Trend component (T_t)

In order to model water deficit time series, a null hypothesis of no trend in the series was adopted. Turning point test and Kendall's rank correlation test were performed for detection of the trend in the time series. In this analysis, annual water deficit data have been used for detecting the trend. The annual series has been assumed to give better results for the trend component and suppresses the effect of periodic components in the series (Kottegoda, 1980). The details of these tests are presented in Clarke (1984). If the trend is present, then T_t was removed by regression.

Periodic component (P_t)

The periodic component in a time series is deterministic in nature having the property to repeat itself at regular intervals. The existence of P_t was first identified by the serial correlogram, i.e., a graph of autocorrelation coefficients, r_k against lag k.

Since a periodic time series P_t, usually is not stationary, it was then expanded into a Fourier series representation expressed as:

$$P_t = A_0 + \sum_{K=1}^{\infty}\left[A_K \text{Cos}\left(\frac{2\pi Kt}{p}\right) + B_K \text{Sin}\left(\frac{2\pi Kt}{p}\right)\right] \quad .. \ (2)$$

where,

$$A_0 = \frac{1}{N}\sum_{i=1}^{N} X_t \quad \ldots (3)$$

$$A_K = \frac{2}{N}\sum_{i=1}^{N} X_t \text{Cos}\left(\frac{2\pi Kt}{p}\right) \quad \ldots (4)$$

$$B_K = \frac{2}{N}\sum_{i=1}^{N} X_t \text{Sin}\left(\frac{2\pi Kt}{p}\right) \quad \ldots (5)$$

in which K = number of significant harmonics; t = time interval within the year; N = number of observation points; p = base period; and A_K and B_K = Fourier series coefficients.

when p is even, then

$$A_M = \frac{1}{N}\sum_{i=1}^{N} X_t \text{Cos}\left(\frac{2\pi Kt}{p}\right) \quad \ldots (6)$$

$$B_M = 0 \quad \ldots (7)$$

These coefficients were obtained by a least square fit of the data to the K[th] harmonics components and then a least square approximation was given by the finite series:

$$P_t = A_O + \sum_{K=1}^{M}\left[A_K \text{Cos}\left(\frac{2\pi Kt}{p}\right) + B_K \text{Sin}\left(\frac{2\pi Kt}{p}\right)\right] (8)$$

where, M is the number of significant harmonics (maximum, p/2). For later use, it was more convenient to use the alternate form for P_t given as under:

$$P_t = A_O + \sum_{t=1}^{M} D_K \text{Cos}\left(\frac{2\pi Kt}{p} - \theta_K\right) \quad \ldots. (9)$$

where,

$$D_K = \sqrt{A_K^2 + B_K^2} \quad \ldots (10)$$

and

$$\theta_K = \text{Arc}\tan\left(\frac{B_K}{A_K}\right) \quad \ldots. (11)$$

In Equation (9) if M→ ∞, P_t →X_t then X_t can be represented satisfactory by Equation (2) only. However, it may not be practical or desirable to allow thecondition M→ ∞.

Thus the appropriate approach would be the selection of M, which contains only those harmonics, which are significantly contributing towards X_t. With this as the objective, Analysis of variance, Fourier decomposition of mean square and Cumulative periodogram tests were carried out for determination and selection of the significant harmonic coefficients, A_K and B_K.

β_k ***Analysis of variance test***

The coefficients were tested through the analysis of variance for half the base period in order to obtain the F-ratios. In this analysis, null hypothesis was that the variance explained by a harmonic I, which is (N/2) ($\alpha_k^{2}+ \;^{2}$), where N is the total sample size, is zero. F ratio was found out by mean squared values divided by unexplained variance, if the null hypothesis is not rejected, the sum of squares is added to the residual sum of squares. If the value of F-ratio has been found less than the F-distribution table value at P = 0.05 level of significance, the corresponding harmonic was selected.

Fourier decomposition of mean square test

The contribution of the individual harmonics, towards the mean square, was calculated and the number of harmonics that were dominantly contributing to mean square were selected as the significant harmonics.

Cumulative periodogram test

A graphical method was employed for selecting the significant harmonics in Fourier series fit of a periodic estimate. A graph was drawn between P_i and number of harmonics for selecting the significant harmonics. The

Table 1: Statistical characteristics of weekly water deficit series at SK Nagar (1974 -01)

Week	Mean, mm week-1	SD, mm week-1	CV, %	Skewness	Kurtosis	Week	Mean, mm week-1
1	17.68	2.76	15.60	0.29	0.89	27	29.00
2	18.59	3.65	19.64	-0.48	-0.39	28	17.78
3	18.83	3.35	17.76	-0.38	1.41	29	9.87
4	20.30	2.66	13.12	0.01	2.46	30	9.49
5	20.54	4.70	22.89	-1.51	5.24	31	6.97
6	22.75	2.48	10.88	0.82	-0.21	32	7.67
7	24.51	4.12	16.82	-1.52	6.76	33	7.70
8	27.60	4.00	14.49	0.60	0.77	34	10.47
9	28.48	4.51	15.84	-2.02	8.57	35	11.14
10	31.61	3.66	11.56	0.81	1.34	36	15.19
11	35.69	3.86	10.81	-0.57	0.21	37	15.72
12	37.60	3.75	9.96	-0.39	1.10	38	18.91
13	39.18	3.98	10.15	-0.41	-0.84	39	18.75
14	43.41	6.25	14.41	-0.17	0.49	40	20.00
15	45.31	7.31	16.14	1.72	5.26	41	22.55
16	47.22	5.92	12.54	0.15	-0.29	42	22.40
17	47.83	11.67	24.40	-2.21	5.65	43	20.10
18	50.55	6.71	13.27	-0.01	1.68	44	20.33
19	49.16	15.04	30.59	-2.66	7.34	45	17.60
20	50.73	10.80	21.30	-2.55	9.12	46	15.54
21	51.96	7.18	13.81	-1.07	0.42	47	16.41
22	51.70	10.10	19.54	-1.85	4.11	48	17.36
23	49.82	14.15	28.41	-2.16	4.97	49	15.86
24	43.80	23.91	54.60	-0.05	1.42	50	16.10
25	38.12	21.65	56.79	-0.78	-0.83	51	15.65
26	35.34	21.40	60.55	-0.75	-0.79	52	16.81

Table 2: Fourier series coefficients for weekly water deficit

Order	Alpha	Beta	Amplitude	Expl. var.	Cum. expl var	Order	Alpha	Beta
1	13.97	-7.00	15.62	63.55	63.55	14	0.21	0.48
2	-9.85	-2.79	10.23	27.27	90.82	15	-0.22	-0.05
3	4.51	-0.83	4.59	5.49	96.31	16	0.11	-0.23
4	-1.40	1.36	1.97	1.00	97.31	17	-0.32	-0.26
5	-0.92	-0.31	0.97	0.24	97.56	18	0.26	-0.07
6	-0.56	0.37	0.67	0.12	97.68	19	0.05	0.08
7	-0.04	-0.32	0.33	0.03	97.70	20	-0.01	0.09
8	0.21	-0.11	0.24	0.02	97.72	21	0.05	-0.10
9	-0.07	-0.37	0.38	0.04	97.76	22	-0.09	-0.14
10	0.56	0.34	0.66	0.11	97.87	23	0.02	-0.21
11	-0.20	-0.30	0.42	0.05	97.91	24	-0.15	-0.11
12	0.33	0.20	0.38	0.04	97.95	25	0.05	-0.04
13	-0.33	-0.19	0.38	0.04	97.99	26	0.00	0.32

significant harmonics were selected up to the fast increase in P_i and the rest of harmonics were rejected. The periodic component was then removed from the time series using the harmonic constants. The remainder being random was applied to the stochastic component.

Stochastic component (S_t)

A stochastic model of the form of autoregressive model (AR), was used for the presentation of the time series. An autoregressive model of order p, AR(p) can mathematically

Table 3: Analysis of variance of weekly water deficit

S.No.	Harmonic	Degree of freedom	Sum of Squares	Mean squares	F_{cal}	F_{tab}	
						0.01	0.05
1	5,6,---,26	43	2523.0	58.7	0.19	1.57	1.38
2	Residual	1412	429385.4	304.1			
3	4	3	2813.2	937.7	3.17	3.78	2.6
4	Residual	1452	429095.1	295.5			
1	4,5, ---,26	45	5336.2	118.6	0.39	1.56	1.37
2	Residual	1410	426572.2	302.5			
3	3	2	15339.0	7669.5	26.75	4.61	3
4	Residual	1453	416569.3	286.7			
1	3,4, ---,26	47	20675.2	439.9	1.51	1.55	1.37
2	Residual	1408	411233.2	292.1			
3	2	2	76243.8	38121.9	155.74	4.61	3
4	Residual	1453	355664.6	244.8			
1	2,3, ---,26	49	96919.0	1977.9	8.30	1.54	1.36
2	Residual	1406	334989.4	238.3			
3	1	2	177647.4	88823.7	507.59	4.61	3
4	Residual	1453	254261.0	175.0			

be expressed as:

$$S_t = \sum_{k=1}^{p} \phi_{p,k} S_{t-k} + a_t \quad \ldots (12)$$

where, $\phi_{p,k}$ = the autoregressive model parameters; k = the number of parameters, k = 1, 2, . . ., p; p = the order of the model; and a_t = independent random number

The fitting procedure of the AR (p) model involved three steps, viz. model identification, parameter estimation and model diagnostic checking.

Model identification

For selecting the best model and thereby to estimate the parameters of the model structure residual variance criteria method was used. In this method, residual variance, $S_z^2(p)$ was computed from:

$$S_z^2(p) = \frac{1}{N-2p-1} S(\mu, \alpha_1, \alpha_2, ..., \alpha_p) \quad \ldots (13)$$

in which $S(\mu, \alpha_1, \alpha_2, ..., \alpha_p)$ is known as the residual sum of squares and has been computed from:

$$S(\mu, \alpha_1, \alpha_2, ..., \alpha_p) = (N-p)(C_o - \alpha_1 C_1 - \alpha_2 C_2 - ... - \alpha_p C_p) \quad \ldots (14)$$

where, N = number of observation points; $\alpha_1, \alpha_2, ..., \alpha_p$ = parameters of the corresponding model; and C_o, C_1, C_2, . . . C_p = autocovariance function at lag p, p = 0, 1, 2, . . ., p

The residual variance values were computed for all estimated lag p. The rule for this criterion is to select the model order with minimum value of (Kottegoda, 1980). Based on this criterion, order of the AR model was identified.

Parameter estimation

The autoregression parameters of different orders were estimated before the proper order for AR terms has been identified. In general, the estimates of p[th] order model has been obtained computing following equations recursively.

$$\phi_{p,p} = \left[\frac{r_p - \sum_{k=1}^{p-1} (\phi_{p-1,k})(r_{p-k})}{1 - \sum_{k=1}^{p-1} (\phi_{p-1,k})(r_k)} \right] \quad \ldots (15)$$

and

for k = 1, 2, …, p-1 …(16)

In the estimated parameters, , suffix p and k indicate the order and the number of parameters of AR (p) model.

Table 4: Fourier decomposition of periodic component in weekly water deficit series

Order	A_k	B_k	Amplitude	Theta	Explained variance	Cumulative explained variance
1	-6.400	14.230	15,603	-1.148	41.005	41.005
2	-3.633	-9.543	10.211	1.207	17.563	58.568
3	-0.280	4.522	4.530	-1.509	3.457	62.025
4	1.099	-1.496	1.857	-0.937	0.581	62.605
5	-0.082	0.969	0.973	-1.487	0.159	62.765
6	0.221	-0.571	0.612	-1.202	0.063	62.828
7	-0.271	-0.031	0.273	0.114	0.013	62.840
8	-0.066	0.206	0.216	-1.262	0.008	62.848
9	-0.447	0.066	0.452	-0.146	0.034	62.883
10	0.523	0.398	0.657	0.651	0.073	62.955
11	-0.361	-0.175	0.401	0.452	0.027	62.982
12	0.269	0.139	0.303	0.477	0.015	62.998
13	-0.309	-0.169	0.352	0.499	0.021	63.019
14	0.394	0.000	0.394	0.001	0.026	63.045
15	0.100	-0.217	0.239	-1.137	0.010	63.055
16	-0.189	0.213	0.285	-0.845	0.014	63.068
17	-0.337	-0.124	0.359	0.353	0.022	63.090
18	-0.066	0.271	0.279	-1.331	0.013	63.103
19	0.307	-0.072	0.316	-0.231	0.017	63.120
20	-0.111	0.211	0.239	-1.088	0.010	63.130
21	0.027	-0.049	0.056	-1.075	0.001	63.130
22	-0.117	0.274	0.298	-1.167	0.015	63.145
23	0.321	-0.513	0.605	-1.012	0.062	63.207
24	-0.316	0.271	0.416	-0.708	0.029	63.236
25	-0.089	-0.268	0.282	1.250	0.013	63.249
26	0.767	0.000	0.767	0.000	0.099	63.349

The sum of the periodic and stochastic component forms the generated value of the observed data. The difference was termed as residual, which was tested to check the adequacy of the formulated model.

Model diagnostic checking

The selected model was validated for its suitability through the diagnostic checking. Serial correlation and sum of square analysis were used as a tool for diagnostic checking. In Serial correlation analysis, after fitting the model to stochastic component the residuals were obtained. The serial correlation coefficient of the residuals was estimated for lag k and the correlogram with the 95 % upper and lower confidence limits were drawn. The serial correlation coefficients falling well within the tolerance limits indicate the suitability of the model for the water deficit series and the residuals were assumed to be white noise (random). In sum of squares analysis method sum of squares of residuals and deviation of observed series from their mean value were calculated to get R^2 by ratio and used as a tool to assess the adequacy of the model (Clarke, 1984).

Validation of stochastic model

The major application of modelling time series is to generate or forecast future data. Generating the water deficit time series for the entire sampling period was first checked using the weekly developed water deficit models. Forecasting was made for two years ahead from 2002 to 2003. The generated/forecasted values from each model were compared to the observed data. The variation of the generated/ forecasted and observed series was presented graphically with respect to time. Correlation coefficient of mean generated series and mean observed series was determined. Other statistical parameters of generated and observed series were also computed for validation of the model.

RESULTS AND DISCUSSION

Some of the statistical characteristics of weekly water deficit series used for stochastic modelling for S.K.Nagar are computed and presented in Table 1. The mean weekly water deficit series, as shown in Table 1, varied from 6.97 mm to 51.96 mm. No Large variability among the weekly values of water deficit of different years was observed. This is further confirmed by the lower value of estimated standard deviations (SD). The standard deviation of weekly water deficit ranged from 2.48 mm to 23.91 mm.

Values of coefficient of variation (CV) as presented in

Table 5: Residual variance [Sz^2(p)] values for weekly water deficit series Autocovariance at lag zero, (C_0) = 431908.339

Lag	Auto-correlation	Auto-covariance	Residual variance[Sz^2(p)]	Lag	Auto-correlation
1	0.80	346173.23	154451.56	14	-0.17
2	0.71	307951.51	270219.59	15	-0.21
3	0.62	268343.09	272187.44	16	-0.23
4	0.55	237203.23	272574.27	17	-0.24
5	0.45	194278.66	272960.72	18	-0.25
6	0.35	149942.74	281638.95	19	-0.27
7	0.26	111662.83	285908.19	20	-0.27
8	0.17	74154.96	288572.34	21	-0.27
9	0.10	41610.01	290426.58	22	-0.27
10	0.03	12095.62	291401.83	23	-0.26
11	-0.03	-12628.79	292036.05	24	-0.26
12	-0.08	-34561.46	292704.22	25	-0.26
13	-0.13	-57487.38	294393.06	26	-0.25

Table 6: Model order, parameters and structure of weekly water deficit series

Model order	$\Phi_{(p,k)}$	Values	Mean, mm week^{-1}	Periodic component, mm week^{-1}	Stochastic component, mm week^{-1}
2	$\Phi_{(2,1)}$	0.643	26.23	-6.4Cos(2πt/p)+14.23Sin(2πt/p) -3.63Cos(4πt/p)-9.54Sin(4πt/p) -0.28Cos(6πt/p)+4.52Sin(6πt/p) 1.1Cos(8πt/p)-1.49Sin(8πt/p)	$0.643S_{t-1}+0.197S_{t-2}+a_t$
	$\Phi_{(2,2)}$	0.197			

Table 1 are greater than zero which shows the importance of time variability of weekly water deficit series. Further, the values of CV significantly different than zero indicate that water deficit is mutually dependent.

The skewness coefficient of weekly water deficit, a characterization of the degree of asymmetry of distribution around its mean, ranging from -2.66 to 2.73 indicated that water deficit at SK Nagar was slightly right skewed.

Kurtosis, indicates flatness or peakedness of a distribution of the water deficit series compared with normal distribution (C_K = 3). Positive kurtosis (C_K > 3) indicates a relatively peaked distribution while less than 3 and negative kurtosis, a relatively flat distribution. The results of kurtosis ranged from -1.68 to 9.12 indicating that the series shows a wide variation in weekly water deficit distribution during the study period.

The annual water deficit series obtained from 28 (1974 - 2001) year's water deficit data was used for identification and detection of trend component in the series. The estimated values for turning point (-2.47) and Kendall's rank correlation test (-1.66) were found to be within the acceptable range of ± 2.58 at 0.01 level of significance. Hence, the hypothesis of no trend was not rejected. From this analysis it is confirmed that the trend component in water deficit series is absent and the observed series were found to be trend free.

Periodic component was identified by its cyclic phenomenon imposed on the series and detected through the construction of a correlogram, a graph showing the relationship between the autocorrelation functions on the ordinate and lag K on the abscissa. For proper identification and interpretation of autocorrelogram, the estimate of autocorrelation function for weekly water deficit was made for S.K.Nagar.

The autocorrelation functions for the weekly time series up to lag 104 were determined. Correlogram for the weekly observed series along with the tolerance limit is shown in Fig. 1. It is also seen that the autocorrelogram of the time series falls out of the confidence limits indicating the presence of time dependant series, i.e., $X_{(t+1)}$ is dependent on $X_{(t)}$, and $X_{(t+2)}$ is dependant on $X_{(t+1)}$, and so on. Values of lag one autocorrelation coefficients for the series lies outside the range of confidence limit and is significantly different from zero confirming the mutual dependence in water deficit observed series.

The estimation and removal of periodic component from the time series is done through harmonic analysis. To

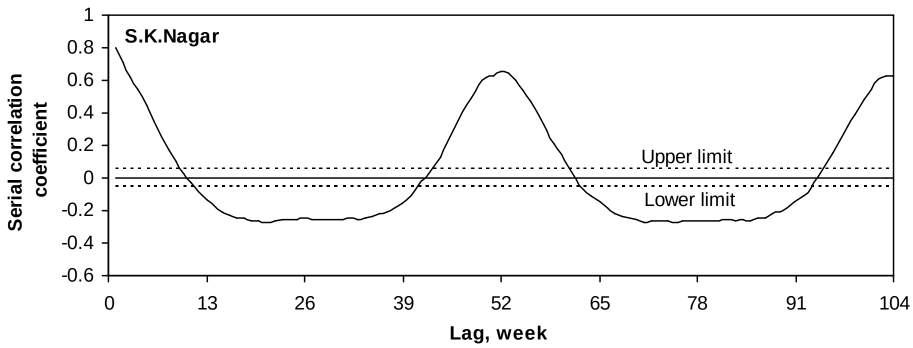

Fig. 1: Correlogram of weekly water deficit

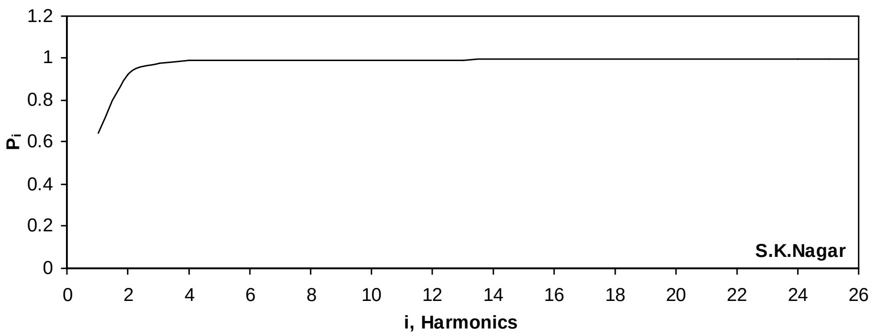

Fig. 2: Cumulative periodogram of weekly water deficit

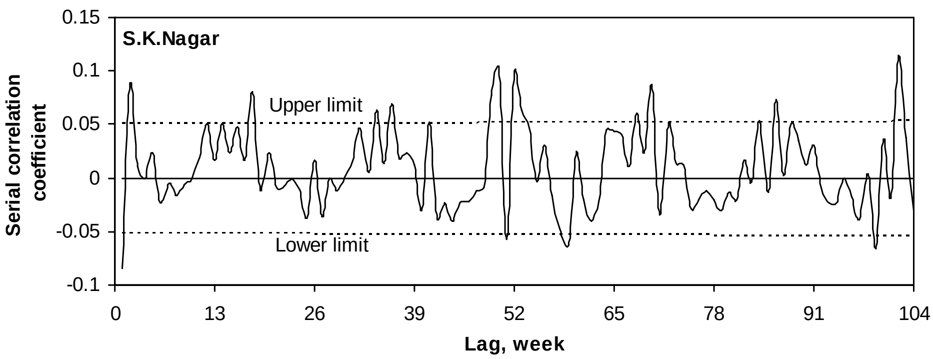

Fig. 3: Correlogram of the residual series of weekly water deficit

estimate the coefficient of harmonics to be fitted in the periodic component, the numbers of harmonics that significantly contribute to periodicities were identified through three different test approaches.

Analysis of variance test computed the estimates of a and b parameters for determination of the number of significant harmonics to be fitted in the periodic series.

Table 2 shows the estimates of the parameters along with amplitude and explained variance for weekly time series. The numbers of significant harmonics have been then detected from the analysis of variance given in Table 3. In this analysis, the parameters a and b were evaluated for all harmonics considered in weekly series to obtain the estimated F-values. The harmonics for which F ratio were greater than the table value of F at 0.05 level of significance have been considered as significant harmonics. The analysis of variance (Table 3) revealed that four harmonics were found to be significant in weekly water deficit series.

In Fourier decomposition of mean square test the numbers of significant harmonics that represent the periodic component were obtained by evaluating the Fourier series coefficients A_k and B_k. The contribution of the individual

Table 7: Statistical characteristics of observed, generated and residual weekly water deficit series

Series	Mean, mm	SD, mm	Skewness	Kurtosis	Variance	ISE
Observed	26.21	17.17	0.35	-0.64	294.85	
Generated	26.23	15.70	0.44	-0.73	246.42	0.0033
Residual	0.00	3.33	-1.11	7.36	11.09	

Table 8: Statistical characteristics of observed, predicted and residual weekly water deficit series

Series	Mean, mm	SD, mm	Skewness	Kurtosis	Variance	ISE
Observed	26.5	16.82	0.18	-0.73	282.79	
Predicted	26.5	15.15	0.37	-0.78	229.45	0.0134
Residual	0.00	3.64	-1.24	6.40	13.27	

harmonics towards the mean square is shown under the explained variance and those harmonics, which dominantly contribute to mean square are selected as the significant harmonics. The results (Table 4) indicated that at SK Nagar first three harmonics have contributed more than 62.02 % to the total variation caused by the periodic component, while only about 1.32 % have been contributed by the rest of the harmonics.

In cumulative periodogram test a graphical procedure, plotting P_i against i, called the cumulative periodogram, has been employed as criteria for obtaining the significant harmonics to be fitted in a periodic component. It is observed from the plot of P_i against i (Fig. 2) that the first four harmonics appeared to be the periodic part of the fast increase in water deficit series.

The three criteria used to identify the number of significant harmonics to be used in modelling periodic component were found to be inconsistent at SK Nagar. Thus, a compromised decision was made to limit the actual number of harmonics to be fitted in the periodic modelling, considering the periodic leak that may occur and thereby to avoid passing on to the stochastic component. Accordingly, first four harmonics were considered in modelling the periodic component of weekly series.

The Fourier series coefficients, A_K and B_K were substituted in the Equation (2) and deterministic periodic components, P_t, have been computed for all values of t, where t is total period, which is 1456 weeks. After determination of the periodic component, the same was then removed by deducting it from the observed time series. The remaining series is a stochastic component part, which is required to be fitted by an autoregressive model of suitable order.

The order of the model was determined by the residual variance procedure explained earlier. The estimated values for weekly series of 26 lags are represented in Table 5. According to the comparison made of the serial autocorrelation of residual series, autoregressive model order two has been selected for weekly water deficit series. The selected model order, parameters, periodic component and stochastic component for the weekly series are given in Table 6. The formulated model structure has been used to generate similar sequenced series of weekly water deficit.

The formulated model was subjected to diagnostic checks to test its adequacy for representing the time series dependent structure of the water deficit series. Sum of squares analysis and auto- correlation analysis were used for diagnostic checking. The required measure value, R^2 of 0.9626 obtained in the sum of squares analysis indicated that the developed model has a best goodness of fit to generate weekly water deficit series. Fig. 3 shows the resulting correlogram of weekly series of autocorrelation analysis. The results show that for all lags the auto-correlation function falls fairly within the confidence limits.

Validation of the model, as shown in Figure 4, was made by comparison of generated with observed water deficit series. Fig. 4 indicates that there is a close agreement between observed and generated water deficit series.

The basic statistical characteristics for the modelling period are given in Table 7. The values of mean, standard deviation, coefficient of skewness, kurtosis and variance show that for observed and generated series statistical characteristics are not significantly different. Also very low integral square error (ISE) shows that the formulated models are significantly adequate for generating weekly water deficit.

The values of correlation coefficient (r) between observed and generated weekly series for the modelled period was worked out to 0.9841. The mean weekly observed water deficit series of period under consideration were also compared with their respective generated series. Fig. 5 depicts the variation between the mean weekly observed and

Fig. 4: Variation of weekly observed and generated water deficit series

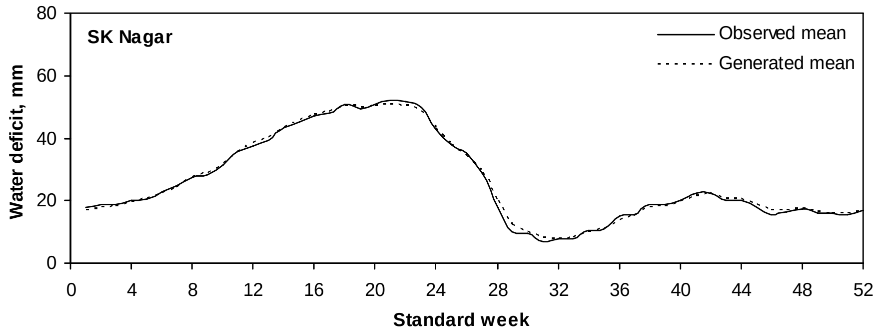

Fig. 5: Variation of mean weekly observed and generated water deficit series

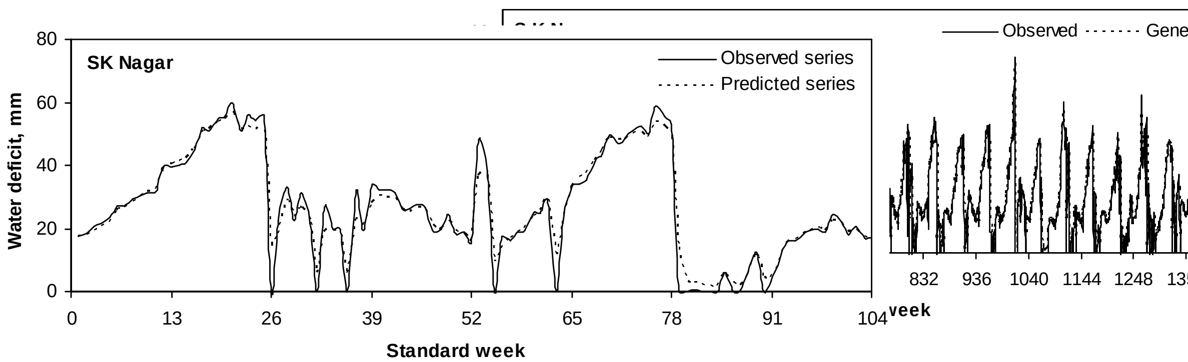

Fig. 6: Variation of observed and predicted weekly water deficit series

generated series of the modelled period.

The value of correlation coefficient r, between the mean observed and generated weekly water deficit series was also worked out as 0.9983. Therefore, the model structure formulated can be used for long-term prediction of weekly water deficit for the station. Similar results were also obtained by Jat *et al*., (2003) for semi-arid region of Rajasthan.

The results of predicted water deficit for weekly series of two years ahead (2002 and 2003) are depicted in Fig. 6. The basic statistical characteristics of observed and predicted weekly series such as mean, standard deviation, coefficient of skewness, kurtosis and variance were also estimated for comparison. As shown in Table 8, the observed and predicted series have not produced significantly different results indicating the adequacy of the model for predicting water deficit for the station. The values of correlation coefficient r, in the weekly series and mean weekly series were observed to be 0.9794 and 0.9870, respectively.

Above results for predicted and observed weekly water deficit series of two years (2002 and 2003) indicate adequacy of the model for predicting water deficit series for respective periods for S.K. Nagar. Values of correlation coefficient and statistical characteristics for observed and predicted series confirm the reliability of weekly water deficit model for generation of data. Therefore the model may be employed to generate the weekly water deficit values, which can be used in planning and operation of irrigation activities.

CONCLUSION

The objective of the study on the stochastic analysis of water deficit is to formulate a mathematical model of the stochastic of water deficit. The study revealed that the developed model is feasible. It was found that the weekly water deficit series were trend free and periodic and stochastic in nature. The significance of the study is to show that the past records of the data provide valuable information for determining the basic time dependent structure of water deficit series.

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