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Sensitivity of Penman–Monteith estimates of reference evapotranspiration to errors in input climatic data

M. H. ALI[1], , A. K. M. ADHAM[2], M. M. RAHMAN[2], A.K. M. R. ISLAM[3]

[1]*Agricultural Engineering Division, Bangladesh Institute of Nuclear Agriculture, P.O. Box – 4, Mymensingh 2200, Bangladesh, Email: mha_bina@yahoo.com.*

[2] *Dept. of Irrigation & Water Management, Bangladesh Agricultural University, Mymensingh 2202, Bangladesh*

[3] *Graduate Training Institute, Bangladesh Agricultural University, Mymensingh 2202, Bangladesh*

ABSTRACT

The Penman-Monteith (P-M) equation with its new definition of reference crop evapotranspiration (ET_0) is recommended by FAO as the standard method of crop water requirement calculation, and also to compare other methods. The ET_0 component of the CROPWAT model, which is based on the P-M equation, was examined for sensitivity to errors in input data under the environment of a semi-humid sub-tropic region of Bangladesh. The results showed that the ET_0 estimates are most sensitive to maximum temperature and least sensitive to minimum temperature. The order of sensitivity noticed is: maximum temperature > relative humidity > sunshine duration > wind speed > minimum temperature. The sensitivity coefficients showed seasonal variation. The model parameter 'Angstrom's coefficients' showed sensitivity to errors in single or pair values. The implications of sensitivity to ET_0 estimates and in selecting appropriate method for ET_0 estimation in a data-short environment are discussed.

Keywords: Sensitivity, Penman-Monteith equation, reference evapotranspiration, CROPWAT

The estimation of crop water requirement is one of the principal steps in the planning, design and operation of irrigation and water resources systems. Crop water requirements vary with crop characteristics and local condition. Models for predicting reference evapotranspiration (ET_0) range from deterministically-based combined energy balance - vapor transfer approaches to empirical relationships based on climatological variables, or to evaporation from a standard evaporation pan. Updated procedures for calculating ET_0 were established by FAO. However, there are many regions of the world, specially in developing countries, where only limited meteorological information is available for estimating ET_0. According to FAO-1992 (Smith *et al.,* 1992), the Penman-Monteith method gives more consistent ET_0 estimates and performs better than other ET_0 methods when compared with lysimeter data. FAO-1992 also suggested to compare and validate other methods of ET_0 estimates with respect to Penman–Monteith (P-M) method.

Sensitivity analysis indicates the simulation error that results if an error is made when assuming the original parameter values. Second, sensitivity indicates how changing a characteristic variable can influence the simulation output or system comparisons. Beven (1979) evaluated the sensitivity of the evapo-transpiration (ET) component of SHE (Systeme Hydrologique Europeen) model of catchment hydrology, which is based on Penman-Monteith equation. He found that for dry canopy conditions, the sensitivity of P-M estimates of actual ET to different input data and parameters is very dependent on the values of the aerodynamic and canopy resistance. Piper (1989) examined the sensitivity of Penman estimates of evaporation to errors in input data and to uncertainties in the values of the parameters used in the equation. He noted that the effects of uncertainty in the equation's parameter values were small and comparable to the sensitivities of the other input variables – wet bulb depression, wind run, and sunshine hours. But the estimates were sensitive to temperature.

Scenario of ET_0 estimates of P-M method due to variation/change in input data would be useful in selecting appropriate method for a particular site with limited data set. The objective of this study was to investigate the sensitivity of ET_0 estimates to errors in input data and to uncertainties in the use of the P-M method where a complete set of data is not available. Now-a-days the FAO CROPWAT software, which is based on the P-M equation, is widely used to calculate ET_0. Hence, it was used in this study to calculate ET_0.

MATERIALS AND METHODS

Sensitivity Analysis

The estimate of ET_0 is calculated from a number of input data or variables, any of which may be subject to error. The ET_0 estimate may be expressed in the general form:

$$ET_0 = f(p_1, p_2 .... p_n) \qquad .... \quad (1)$$

where there are n input data variables, p_i. The error ΔET_0 in ET_0, that results from errors in the p_i, can be expressed in the form:

$$ET_0 + \Delta ET_0 = f(p_1 + \Delta p_1, p_2 + \Delta p_2, ... p_n + \Delta p_n) \quad ...... (2)$$

Expanding equation (2) in Taylor series and ignoring second-order terms and above, leads to:

$$\Delta ET_0 = \frac{\partial ET_0}{\partial p_1}\Delta p_1 + \frac{\partial ET_0}{\partial p_2}\Delta p_2 + ... + \frac{\partial ET_0}{\partial p_n}\Delta p_n \quad .... (3)$$

where the differentials $\partial ET_0/\partial p_i$ are the absolute sensitivity of the estimate to each variable, p_i, and Δp_i is the individual error associated with p_i (Beven, 1979). These sensitivity coefficients are themselves sensitive to the relative magnitudes of ET_0 and the p_i.

McCuen (1974) described the sensitivity of a system output, in this case ET_0, to variation in system parameters or variables (p_i) as the mathematical derivative of $\partial ET_0/\partial_{pi}$. A non-dimensional relative sensitivity is given by:

$$S_i = \frac{\partial ET_0}{\partial p_i} \cdot \frac{p_i}{ET_0} \quad .... (4)$$

The sensitivity coefficient S_i now represents the fraction of the change in p_i that is transmitted through to the estimate of ET_0. Coleman and DeCoursey (1976) suggested an alternative form:

$$S_i^* = \frac{\partial ET_0}{\partial p_i}\left(\frac{p_i - p_{i0}}{ET_0}\right) \quad .... (5)$$

where p_{i0} is the minimum value that p_i can assume. They stated that this form of the equation is more meaningful when comparing variables some of which may have a range in variability quite different from their numerical value.

The sensitivity of the estimate of ET_0 to error or uncertainties in the input data namely, maximum temperature (Tmax), minimum temperature (Tmin), average relative humidity (RH), wind speed (WS), and sunshine hour (SH) were calculated for the following coefficients:

S(Tmax): sensitivity to maximum air temperature

S(Tmin): sensitivity to minimum air temperature

S(RH): sensitivity to average relative humidity

S(WS): sensitivity to wind speed

S(SH): sensitivity to sunshine hour

Another set of sensitivity coefficients were also tested to represent the uncertainties in the values of the parameters used in the P-M equation, the Angstrom's coefficients. The sensitivity coefficients are *S(Ang-a)* and *S(Ang-b)*, respectively. In this study, the sensitivity coefficients have been calculated using equation (4) and (5).

Data

The selection of stations were made covering different agro-ecological zones of Bangladesh. Data for the climatic variables were collected from Bangladesh Meteorological Department. Data for different seasons (wet/summer and dry/winter) were tested to observe if any difference between the seasonal distribution of the sensitivity coefficients existed. Station details are given in Table-1. The monthly mean meteorological data at the stations used in this study are given in Table-2. For the sensitivity coefficient defined in equation (5), the minimum values of the climatic variables (p_{io}) considered in this study are as follows:

Maximum temperature	:	$10\,^0$C
Minimum temperature	:	$0\,^0$C
Relative humidity	:	30 %
Wind speed	:	10 km/d
Sunshine duration	:	0 hr

Thus the S_i & S_i* are the same for minimum temperature and sunshine duration.

Calculation of ET_0

The ET_0 values were calculated using FAO CROPWAT 4, Windows version 4.2, which uses Penman-Monteith equation to calculate ET_0. Each input variable was varied in turn, the other variables remaining unchanged. The studied variation in inputs are shown in Table-3.

The form of P-M equation used in CROPWAT

The Penman-Monteith (P-M) equation is reduced in the form (Smith *et al.,* 1992):

$$ET_0 = \frac{0.0864}{\lambda} \cdot \frac{\Delta(R_n - G) + c_p \rho_a DPV / r_a}{\Delta + \gamma(1 + r_c / r_a)} \quad .... (6)$$

where λ is the latent heat of vaporization ($MJkg^{-1}$); Δ the slope of the vapor pressure versus temperature curve (kPa $^0C^{-1}$); γ the psychrometric constant (kPa $^0C^{-1}$); R_n the net radiation (Wm^{-2}); G the soil heat flux (Wm^{-2}); c_p the specific heat of air (1013 Jkg^{-1} C^{-1}); ρ_a the

Table 1: Details of the meteorological stations from where input weather variables were taken

Sr No.	Station name	Country	Location		Altitude
			Latitude	Longitude	
1	Mymensingh	Bangladesh	$24^0 45'$ N	$90^0 24'$ E	19 m
2	Rajshahi	Bangladesh	$24^0 24'$ N	$88^0 48'$ E	34 m
3	Rangpur	Bangladesh	$25^0 45'$ N	$89^0 15'$ E	34 m

Table 2: Monthly mean meteorological data (standard) for the stations used in the study

Climatic variables	Mymensingh		Rajshahi		Rangpur
	Dry/ Winter	Wet / Summer	Dry/ Winter	Wet / Summer	Dry/ Winter
Max. temperature (^{0}C)	25.43	32.22	25	34	20.4
Min. temperature (^{0}C)	12.93	25.95	13	25.8	9.3
Air humidity (%)	78	84	76	84	83
Wind speed (km/d)	83.52	187.2	84	108	101
Sunshine duration (hr)	6.87	5.98	8	7.5	5.1

Table 3: Variability of the input variables and model parameters studied for sensitivity test

Sr. No.	Climatic variables / coefficients		Variations
1	Max. temperature		± 1, 2, 3, 4, 5, 10, 20 , 30 and 50 percent
2	Min. temperature		± 1, 2, 3, 4, 5, 10, 20 , 30 and 50 percent
3	Relative humidity		± 1, 2, 3, 4, 5, 10, 20 and 30 percent
4	Wind speed		± 1, 5, 10, 20 and 30 percent
5	Sunshine hour		± 1, 2, 3, 4, 5, 10, 20, 30, and 50 percent
6	Angstrom's coefficients	***a***	± 4, 8 and 12 percent
		b	± 2, 4, 6 and 8 percent

atmospheric density (kgm^{-3}); DPV the vapour pressure deficit (kP$_a$); r_a the aerodynamic resistance (sm^{-1}); r_c the bulk canopy resistance (sm^{-1}); and the ratio 0.0864/λ was used to transform Wm^{-2} to mm per day.

RESULTS AND DISCUSSION

Sensitivity of climatic variables

The sensitivity coefficients (S_i , S_i*) of different climatic variables at three locations are summarized in Table 4. The mean sensitivity coefficients of a particular variable exhibited a similar pattern over all locations, except minimum temperature. The sensitivity coefficients all exhibited a seasonal variation. All coefficients but for wind speed exhibited higher values in the 'dry & cold' period (January) than in the 'wet & hot' period (September). Variation of climatic variables in the negative direction responded differently in sensitivity – normally smaller values of sensitivity than that of the positive direction. It may be partly due to the fact that the sensitivity coefficients (in equation 4 & 5) are still sensitive to the values of ET_0 and p_i.

Among the climatic variables, maximum temperature showed the highest sensitivity at all locations. The Fig-1a to 1c confirm this trend. The minimum temperature exhibited a peculiar pattern, sometimes variation in reverse direction. This may be due to interaction of variables, which is not considered in this study. The relative humidity exhibited the second highest sensitivity at Mymensingh & Rajshahi, and showed a consistent pattern. Sunshine duration showed the second highest sensitivity at Rangpur only, and the pattern is not consistent or regular. Considering all the locations and both the directions (+ve & -ve increase), sunshine duration showed the third highest and wind speed showed the forth highest sensitivity. It is known from the physical features that after a critical upper limit of the wind speed, there is no effect of further increase in wind speed (i.e. removal of water vapor from liquid surface). Minimum temperature exhibited the lowest sensitivity value, thus is less sensitive in ET_0 estimation. This is supported by the data shown in Fig.1a & Fig. 1b.

Table 4: Sensitivity coefficients (S_i and S_i^*) of different variables at wet and dry weather condition

Weather variable	Form of sensitivity	Changing direction	Mymensingh		Rajshahi		Rangpur
			Dry	Wet	Dry	Wet	Dry
Maximum temperature	S_i	+ ve	0.808	0.952	0.770	0.775	0.810
		- ve	0.602	0.59	0.599	0.634	0.568
	S_i^*	+ ve	0.527	0.694	0.497	0.575	0.460
		- ve	0.331	0.382	0.335	0.435	0.247
Minimum temperature	S_i	+ ve	0.032	0.111	0.089	0.178	-0.003
		- ve	-0.002	-0.009	0.008	0.127	-0.079
	S_i^*	+ ve	0.032	0.111	0.089	0.178	-0.003
		- ve	-0.002	-0.009	0.008	0.127	-0.079
Relative humidity	S_i	+ ve	-0.39	-0.904	-0.352	-0.382	-0.710
		- ve	-0.286	-0.664	-0.242	-0.262	-0.585
	S_i^*	+ ve	-0.252	-0.595	-0.225	-0.253	-0.470
		- ve	-0.163	-0.397	-0.132	-0.156	-0.352
Wind speed	S_i	+ ve	0.184	0.107	0.198	0.063	0.320
		- ve	0.164	0.086	0.141	0.064	0.123
	S_i^*	+ ve	0.173	0.102	0.119	0.058	0.290
		- ve	0.141	0.08	0.141	0.056	0.109
Sunshine hour	S_i	+ ve	0.289	0.338	0.338	0.473	0.370
		- ve	0.281	0.28	0.280	0.414	0.190
	S_i^*	+ ve	0.289	0.338	0.338	0.473	0.370
		- ve	0.281	0.28	0.280	0.414	0.190

Note: S_i and S_i^* are defined by equation (4) & (5), respectively.

Table 5: Response of Angstrom's coefficients to ET_0 estimates under different pairs

Characteristics	Pairs		ET_0 value (mm d^{-1})
	value of '*a*'	value of '*b*'	
Higher '*a*' value coupled with lower '*b*' value	0.25	0.50	1.97
	0.26	0.49	1.99
	0.27	0.48	2.02
Lower '*a*' value coupled with higher '*b*' value	0.24	0.51	1.95
	0.23	0.52	1.93
	0.22	0.53	1.91

Sensitivity of Angstrom's coefficients

The Fig. 2 illustrates the sensitivity of the model parameter 'Angstrom's coefficients'(for Mymensingh location). The parameter '*b*' showed higher sensitivity with fixed '*a*' than parameter '*a*' with fixed '*b*'. For both cases, the sensitivity decreased with decreasing values of the parameters.

When the sum of the coefficients is equal to 0.75 (i. e. $a+b = 0.75$), the response of different pairs of the parameters to ET_0 estimates are summarized in Table 5. Increasing values of '*a*' coupled with decreasing values of '*b*' increased the ET_0 value. This means that if the value '*b*' is assumed lower than its actual value, the ET_0 will be overestimated. The reverse result applies for a lower '*a*' value coupled a with higher '*b*' value.

Implications in ET_0 estimates

The percentage change of ET_0 to percentage error in input data, and the sensitivity coefficients are useful to consider the absolute errors or uncertainties that might be expected. The ET_0 prediction is highly sensitive to the values of maximum temperature, meaning that accurate data for this factor or variable is important. Similarly, the relative humidity is the second most and sunshine duration is the third most important factor. Wind speed and minimum temperature are comparatively less important. So that the areas where wind speed data are not available, interpolation or gauging of the parameter from nearby stations, or use of historical average values may be used to estimate ET_0 using the P–M method or the CROPWAT model.

In the context of calculating crop water requirements,

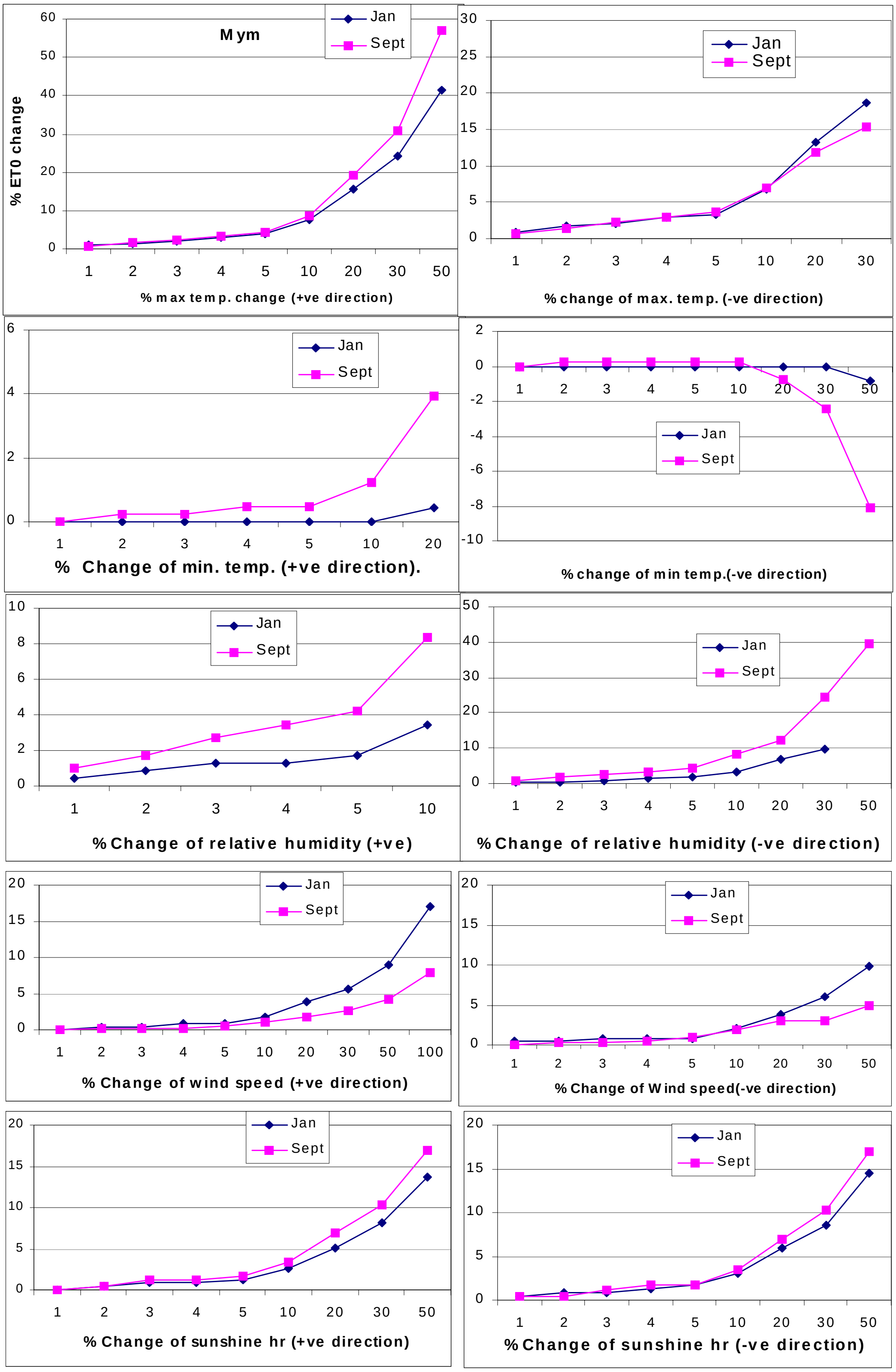

Fig.1a: Change of climatic variables (%) vs corresponding change in ET_0 values (%) (Mymensingh)

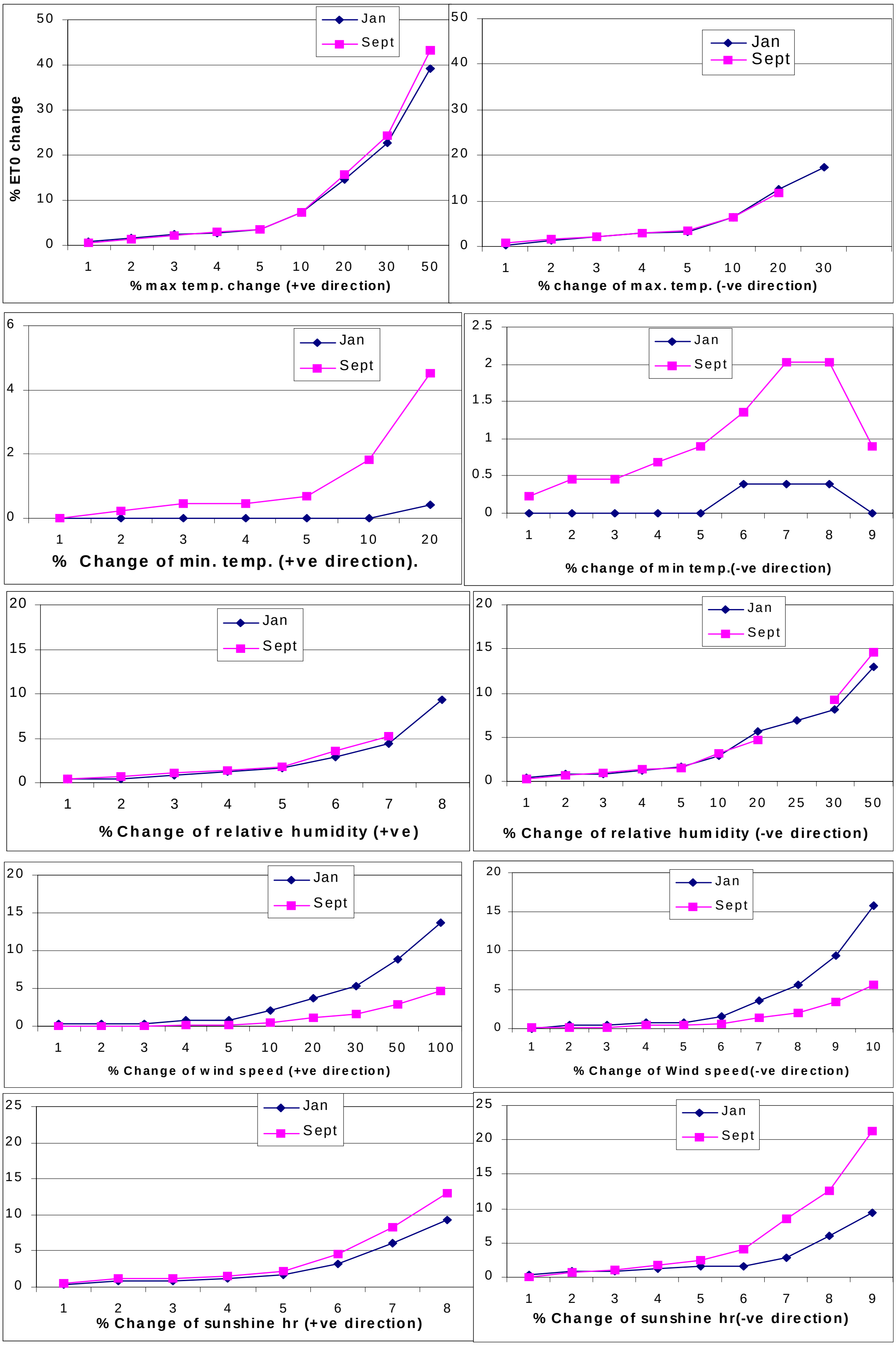

Fig. 1b: Change of climatic variables (%) vs corresponding change in ET_0 values (%) (Rajshahi)

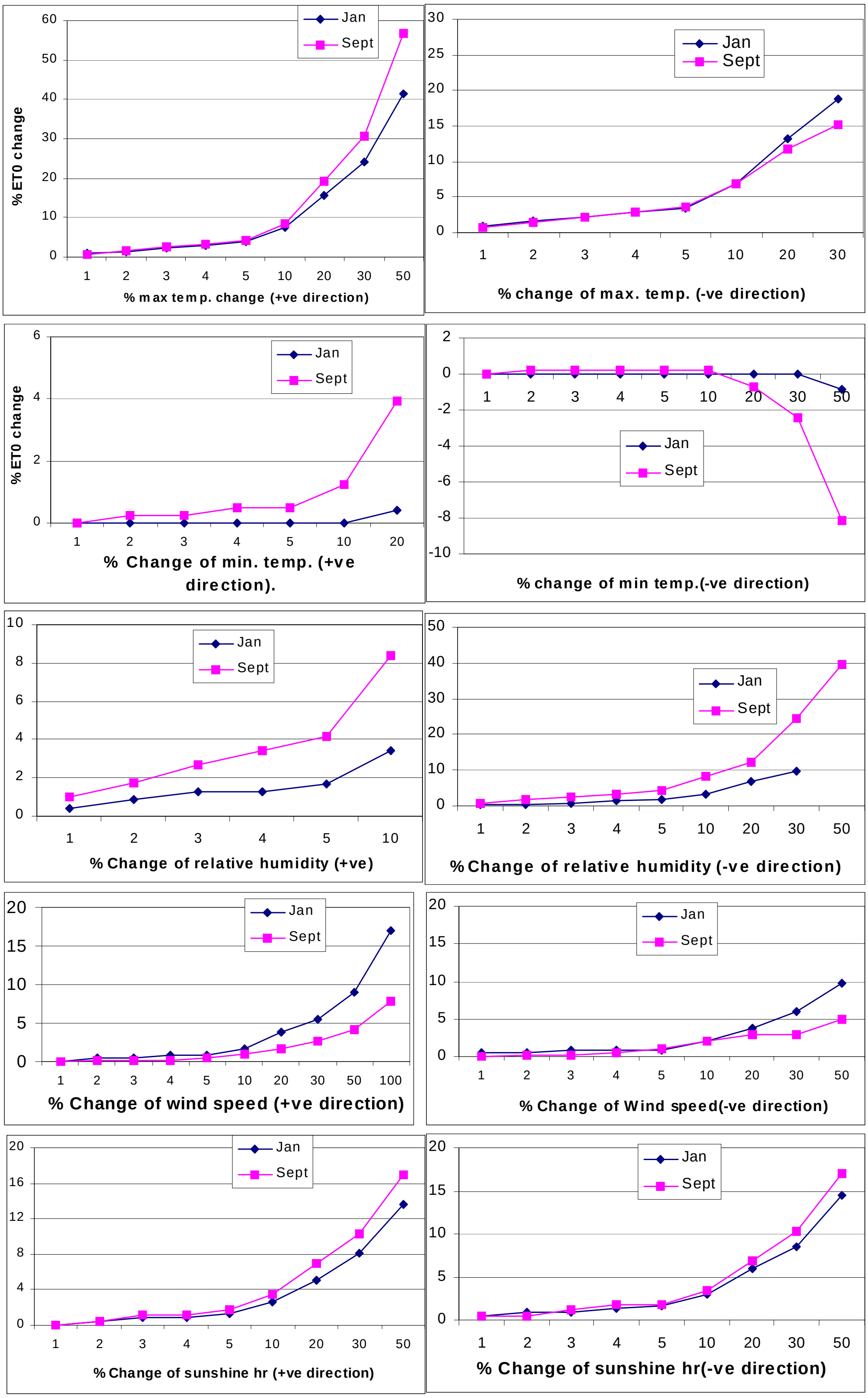

Fig.1c: Change of climatic variables (%) vs corresponding change in ET_o values (%) (Rangpur)

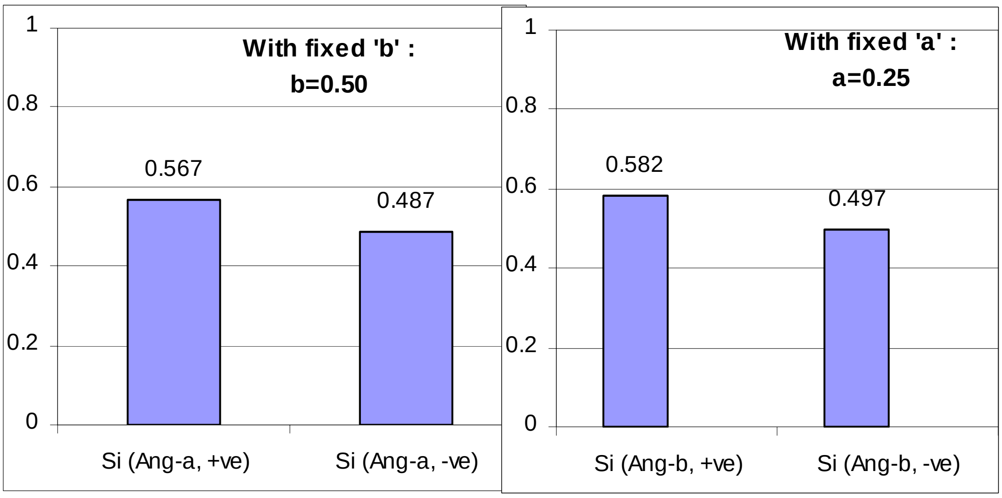

Fig. 2: Sensitivity of Angstrom coefficients (with the data of Mymensingh, dry period)

the accurate measurement of maximum temperature is particularly important. In selecting the appropriate method for ET_0 estimation in a data short environment, the wind speed data may be interpolated or gauged from other locations or historical average values may used for the P-M method or CROPWAT model.

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