# Sensitivity of Penman–Monteith estimates of reference evapotranspiration to errors in input climatic data

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#### **ABSTRACT**

The Penman-Monteith (P-M) equation with its new definition of reference crop evapotranspiration ( $\mathrm{ET_0}$ ) is recommended by FAO as the standard method of crop water requirement calculation, and also to compare other methods. The  $\mathrm{ET_0}$  component of the CROPWAT model, which is based on the P-M equation, was examined for sensitivity to errors in input data under the environment of a semi-humid sub-tropic region of Bangladesh. The results showed that the  $\mathrm{ET_0}$  estimates are most sensitive to maximum temperature and least sensitive to minimum temperature. The order of sensitivity noticed is: maximum temperature > relative humidity > sunshine duration > wind speed > minimum temperature. The sensitivity coefficients showed seasonal variation. The model parameter 'Angstrom's coefficients' showed sensitivity to errors in single or pair values. The implications of sensitivity to  $\mathrm{ET_0}$  estimates and in selecting appropriate method for  $\mathrm{ET_0}$  estimation in a data-short environment are discussed.

Keywords: Sensitivity, Penman-Monteith equation, reference evapotranspiration, CROPWAT

The estimation of crop water requirement is one of the principal steps in the planning, design and operation of irrigation and water resources systems. Crop water requirements vary with crop characteristics and local Models for condition. predicting evapotranspiration (ET<sub>0</sub>) range from deterministically-based combined energy balance - vapor transfer approaches to empirical relationships based on climatological variables, or to evaporation from a standard evaporation pan. Updated procedures for calculating ET<sub>0</sub> were established by FAO. However, there are many regions of the world, specially in developing countries, where only limited meteorological information is available for estimating ET<sub>0</sub>. According to FAO-1992 (Smith et al., 1992), the Penman-Monteith method gives more consistent ET<sub>0</sub> estimates and performs better than other ET<sub>a</sub> methods when compared with lysimeter data. FAO-1992 also suggested to compare and validate other methods of ET<sub>0</sub> estimates with respect to Penman–Monteith (P-M) method.

Sensitivity analysis indicates the simulation error that results if an error is made when assuming the original parameter values. Second, sensitivity indicates how changing a characteristic variable can influence the simulation output or system comparisons. Beven (1979) evaluated the sensitivity of the evapo-transpiration (ET) component of SHE (Systeme Hydrologique Europeen) model of catchment hydrology, which is based on Penman-Monteith equation. He found that for dry canopy conditions, the sensitivity of P-M estimates of actual ET to different input data and

parameters is very dependent on the values of the aerodynamic and canopy resistance. Piper (1989) examined the sensitivity of Penman estimates of evaporation to errors in input data and to uncertainties in the values of the parameters used in the equation. He noted that the effects of uncertainty in the equation's parameter values were small and comparable to the sensitivities of the other input variables – wet bulb depression, wind run, and sunshine hours. But the estimates were sensitive to temperature.

Scenario of  $\mathrm{ET_0}$  estimates of P-M method due to variation/change in input data would be useful in selecting appropriate method for a particular site with limited data set. The objective of this study was to investigate the sensitivity of  $\mathrm{ET_0}$  estimates to errors in input data and to uncertainties in the use of the P-M method where a complete set of data is not available. Now-a-days the FAO CROPWAT software, which is based on the P-M equation, is widely used to calculate  $\mathrm{ET_0}$ . Hence, it was used in this study to calculate  $\mathrm{ET_0}$ .

## MATERIALS AND METHODS

# Sensitivity Analysis

The estimate of  $ET_0$  is calculated from a number of input data or variables, any of which may be subject to error. The  $ET_0$  estimate may be expressed in the general form:

$$ET_0 = f(p_1, p_2, \dots, p_n)$$
 .....(1)

where there are n input data variables,  $p_i$ . The error  $\Delta ET_o$  in  $ET_o$ , that results from errors in the  $p_i$ , can be expressed in the form:

$$ET_0 + \Delta ET_0 = f(p_1 + \Delta p_1, p_2 + \Delta p_2, \dots, p_n + \Delta p_n)$$

Expanding equation (2) in Taylor series and ignoring second-order terms and above, leads to:

where the differentials  $\partial ET_{o}/\partial p_{i}$  are the absolute sensitivity of the estimate to each variable,  $p_{i}$ , and  $\Delta p_{i}$  is the individual error associated with  $p_{i}$  (Beven, 1979). These sensitivity coefficients are themselves sensitive to the relative magnitudes of  $ET_{o}$  and the  $p_{i}$ .

McCuen (1974) described the sensitivity of a system output, in this case  $ET_0$ , to variation in system parameters or variables  $(p_i)$  as the mathematical derivative of  $\partial ET_0/\partial_{pi}$ . A non-dimensional relative sensitivity is given by:

$$S_i = \frac{\partial ET_0}{\partial p_i} \cdot \frac{p_i}{ET_0} \tag{4}$$

The sensitivity coefficient  $S_i$  now represents the fraction of the change in  $p_i$  that is transmitted through to the estimate of  $ET_0$ . Coleman and DeCoursey (1976) suggested an alternative form:

where  $p_{i0}$  is the minimum value that  $p_i$  can assume. They stated that this form of the equation is more meaningful when comparing variables some of which may have a range in variability quite different from their numerical value.

The sensitivity of the estimate of  $ET_0$  to error or uncertainties in the input data namely, maximum temperature (Tmax), minimum temperature (Tmin), average relative humidity (RH), wind speed (WS), and sunshine hour (SH) were calculated for the following coefficients:

S(Tmax): sensitivity to maximum air temperature S(Tmin): sensitivity to minimum air temperature S(RH): sensitivity to average relative humidity

S(WS): sensitivity to wind speed S(SH): sensitivity to sunshine hour

Another set of sensitivity coefficients were also tested to represent the uncertainties in the values of the parameters used in the P-M equation, the Angstrom's coefficients. The sensitivity coefficients are S(Ang-a) and S(Ang-b), respectively. In this study, the sensitivity coefficients have been calculated using equation (4) and (5).

#### Data

The selection of stations were made covering different agro-ecological zones of Bangladesh. Data for the climatic variables were collected from Bangladesh Meteorological Department. Data for different seasons (wet/summer and dry/winter) were tested to observe if any difference between the seasonal distribution of the sensitivity coefficients existed. Station details are given in Table-1. The monthly mean meteorological data at the stations used in this study are given in Table-2. For the sensitivity coefficient defined in equation (5), the minimum values of the climatic variables ( $p_{io}$ ) considered in this study are as follows:

$$\Delta ET = \begin{bmatrix} \text{Maximum temperature} : & 10\,^{\circ}\text{ C} \\ \text{Minimum temperature} : & 0\,^{\circ}\text{ C} \\ \frac{\text{Relative humidiff}T_{0}}{\text{Wind speed}} + \frac{10\,^{\circ}\text{ Relative}}{2p_{2}} + \frac{30\,^{\circ}\text{ N}}{10\,^{\circ}\text{ km/d}} + \frac{2ET_{0}}{2p_{n}} \Delta p_{n} \\ \text{Sunshine duration} : & 0 \text{ hr} \end{bmatrix}$$

Thus the  $S_i \& S_i^*$  are the same for minimum temperature and sunshine duration.

## Calculation of ET

The  $ET_o$  values were calculated using FAO CROPWAT 4, Windows version 4.2, which uses Penman-Monteith equation to calculate  $ET_o$ . Each input variable was varied in turn, the other variables remaining unchanged. The studied variation in inputs are shown in Table-3.

## The form of P-M equation used in CROPWAT

The Penman-Monteith (P-M) equation is reduced in the form (Smith *et al.*, 1992):

$$ET_{o} = \frac{0.0864}{\lambda} \cdot \frac{\Delta(\mathbf{R}_{n} - G) + c_{p}\rho_{a}DPV/r_{a}}{\Delta + \gamma(1 + r_{c}/r_{a})} \dots (6)$$

where  $\lambda$  is the latent heat of vaporization (MJkg<sup>-1</sup>);  $\Delta$  the slope of the vapor pressure versus temperature curve (kPa  $^{0}$ C<sup>-1</sup>);  $\gamma$  the psychrometric constant (kPa  $^{0}$ C<sup>-1</sup>);  $R_{n}$  the net radiation (Wm<sup>-2</sup>); G the soil heat flux (Wm<sup>-2</sup>);  $C_{p}$  the specific heat of air (1013 Jkg<sup>-1</sup> C<sup>-1</sup>);  $\rho_{a}$  the

**Table 1:** Details of the meteorological stations from where input weather variables were taken

| Sr  | Station name | Country    | Locati                            | Location    |      |  |
|-----|--------------|------------|-----------------------------------|-------------|------|--|
| No. |              |            | Latitude                          | Longitude   |      |  |
| 1   | Mymensingh   | Bangladesh | 24 <sup>0</sup> 45 <sup>'</sup> N | 90 ° 24 ′ E | 19 m |  |
| 2   | Rajshahi     | Bangladesh | $24^{\circ}24^{'}$ N              | 88 ° 48 ′ E | 34 m |  |
| 3   | Rangpur      | Bangladesh | 25 ° 45 ′ N                       | 89 ° 15 ′ E | 34 m |  |

**Table 2:** Monthly mean meteorological data (standard) for the stations used in the study

| Climatic variables                  | Mymensingh |        | Rajshahi |        | Rangpur |  |
|-------------------------------------|------------|--------|----------|--------|---------|--|
|                                     | Dry/       | Wet /  | Dry/     | Wet /  | Dry/    |  |
|                                     | Winter     | Summer | Winter   | Summer | Winter  |  |
| Max. temperature ( <sup>0</sup> C ) | 25.43      | 32.22  | 25       | 34     | 20.4    |  |
| Min. temperature ( <sup>0</sup> C)  | 12.93      | 25.95  | 13       | 25.8   | 9.3     |  |
| Air humidity (%)                    | 78         | 84     | 76       | 84     | 83      |  |
| Wind speed (km/d)                   | 83.52      | 187.2  | 84       | 108    | 101     |  |
| Sunshine duration (hr)              | 6.87       | 5.98   | 8        | 7.5    | 5.1     |  |

Table 3: Variability of the input variables and model parameters studied for sensitivity test

| Sr.<br>No. | Climatic variables / coefficients | Variations                                  |
|------------|-----------------------------------|---|
| 1          | Max. temperature                  | ± 1, 2, 3, 4, 5, 10, 20, 30 and 50 percent  |
| 2          | Min. temperature                  | ± 1, 2, 3, 4, 5, 10, 20, 30 and 50 percent  |
| 3          | Relative humidity                 | ± 1, 2, 3, 4, 5, 10, 20 and 30 percent      |
| 4          | Wind speed                        | ± 1, 5, 10, 20 and 30 percent               |
| 5          | Sunshine hour                     | ± 1, 2, 3, 4, 5, 10, 20, 30, and 50 percent |
| 6          | Angstrom's a                      | ± 4, 8 and 12 percent                       |
|            | coefficients b                    | $\pm 2, 4, 6$ and 8 percent                 |

atmospheric density (kgm<sup>-3</sup>); DPV the vapour pressure deficit (kP<sub>a</sub>);  $r_a$  the aerodynamic resistance (sm<sup>-1</sup>);  $r_c$  the bulk canopy resistance (sm<sup>-1</sup>); and the ratio  $0.0864/\lambda$  was used to transform Wm<sup>-2</sup> to mm per day.

## RESULTS AND DISCUSSION

#### Sensitivity of climatic variables

The sensitivity coefficients  $(S_i, S_i^*)$  of different climatic variables at three locations are summarized in Table 4. The mean sensitivity coefficients of a particular variable exhibited a similar pattern over all locations, except minimum temperature. The sensitivity coefficients all exhibited a seasonal variation. All coefficients but for wind speed exhibited higher values in the 'dry & cold' period (January) than in the 'wet & hot' period (September). Variation of climatic variables in the negative direction responded differently in sensitivity – normally smaller values of sensitivity than that of the positive direction. It may be partly due to the fact that the sensitivity coefficients (in equation 4

& 5) are still sensitive to the values of  $ET_0$  and  $p_T$ .

Among the climatic variables, maximum temperature showed the highest sensitivity at all locations. The Fig-1a to 1c confirm this trend. The minimum temperature exhibited a peculiar pattern, sometimes variation in reverse direction. This may be due to interaction of variables, which is not considered in this study. The relative humidity exhibited the second highest sensitivity at Mymensingh & Rajshahi, and showed a consistent pattern. Sunshine duration showed the second highest sensitivity at Rangpur only, and the pattern is not consistent or regular. Considering all the locations and both the directions (+ve & -ve increase), sunshine duration showed the third highest and wind speed showed the forth highest sensitivity. It is known from the physical features that after a critical upper limit of the wind speed, there is no effect of further increase in wind speed (i.e. removal of water vapor from liquid surface). Minimum temperature exhibited the lowest sensitivity value, thus is less sensitive in  $ET_{a}$ estimation. This is supported by the data shown in Fig.1a & Fig. 1b.

**Table 4:** Sensitivity coefficients (S<sub>1</sub> and S<sub>2</sub>\*) of different variables at wet and dry weather condition

| Weather     | Form of          | Changing  | Mymensi | ngh    | Rajs   | hahi   | Rangpur |
|-------------|------------------|-----------|---------|--------|--------|--------|---------|
| variable    | sensitivity      | direction | Dry     | Wet    | Dry    | Wet    | Dry     |
| Maximum     | $S_{i}$          | + ve      | 0.808   | 0.952  | 0.770  | 0.775  | 0.810   |
| temperature |                  | - ve      | 0.602   | 0.59   | 0.599  | 0.634  | 0.568   |
|             | $\mathbf{S_i}^*$ | + ve      | 0.527   | 0.694  | 0.497  | 0.575  | 0.460   |
|             |                  | - ve      | 0.331   | 0.382  | 0.335  | 0.435  | 0.247   |
|             | $S_i$            | + ve      | 0.032   | 0.111  | 0.089  | 0.178  | -0.003  |
| Minimum     |                  | - ve      | -0.002  | -0.009 | 0.008  | 0.127  | -0.079  |
| temperature | $\mathbf{S_i}^*$ | + ve      | 0.032   | 0.111  | 0.089  | 0.178  | -0.003  |
|             |                  | - ve      | -0.002  | -0.009 | 0.008  | 0.127  | -0.079  |
| Relative    | $S_i$            | + ve      | -0.39   | -0.904 | -0.352 | -0.382 | -0.710  |
| humidity    |                  | - ve      | -0.286  | -0.664 | -0.242 | -0.262 | -0.585  |
|             | $\mathbf{S_i}^*$ | + ve      | -0.252  | -0.595 | -0.225 | -0.253 | -0.470  |
|             |                  | - ve      | -0.163  | -0.397 | -0.132 | -0.156 | -0.352  |
| Wind speed  | $S_{i}$          | + ve      | 0.184   | 0.107  | 0.198  | 0.063  | 0.320   |
|             |                  | - ve      | 0.164   | 0.086  | 0.141  | 0.064  | 0.123   |
|             | $S_i^*$          | + ve      | 0.173   | 0.102  | 0.119  | 0.058  | 0.290   |
|             |                  | - ve      | 0.141   | 0.08   | 0.141  | 0.056  | 0.109   |
| Sunshine    | $S_{i}$          | + ve      | 0.289   | 0.338  | 0.338  | 0.473  | 0.370   |
| hour        |                  | - ve      | 0.281   | 0.28   | 0.280  | 0.414  | 0.190   |
|             | $S_i^*$          | + ve      | 0.289   | 0.338  | 0.338  | 0.473  | 0.370   |
|             |                  | - ve      | 0.281   | 0.28   | 0.280  | 0.414  | 0.190   |

Note:  $S_i$  and  $S_i^*$  are defined by equation (4) & (5), respectively.

**Table 5:** Response of Angstrom's coefficients to ET<sub>0</sub> estimates under different pairs

| Characteristics  | Pa           | ET <sub>0</sub> value |                      |
|------------------|--------------|-----------------------|----------------------|
|                  | value of 'a' | value of 'b'          | $(\text{mm d}^{-1})$ |
| Higher 'a' value | 0.25         | 0.50                  | 1.97                 |
| coupled with     | 0.26         | 0.49                  | 1.99                 |
| lower 'b' value  | 0.27         | 0.48                  | 2.02                 |
| Lower 'a' value  | 0.24         | 0.51                  | 1.95                 |
| coupled with     | 0.23         | 0.52                  | 1.93                 |
| higher 'b' value | 0.22         | 0.53                  | 1.91                 |

## Sensitivity of Angstrom's coefficients

The Fig. 2 illustrates the sensitivity of the model parameter 'Angstrom's coefficients' (for Mymensingh location). The parameter 'b' showed higher sensitivity with fixed 'a' than parameter 'a' with fixed 'b'. For both cases, the sensitivity decreased with decreasing values of the parameters.

When the sum of the coefficients is equal to 0.75 (i. e. a+b=0.75), the response of different pairs of the parameters to  $ET_o$  estimates are summarized in Table 5. Increasing values of 'a' coupled with decreasing values of 'b' increased the  $ET_o$  value. This means that if the value 'b' is assumed lower than its actual value, the  $ET_o$  will be overestimated. The reverse result applies for a lower 'a' value coupled a with higher 'b' value.

## Implications in ET<sub>0</sub> estimates

The percentage change of  $ET_o$  to percentage error in input data, and the sensitivity coefficients are useful to consider the absolute errors or uncertainties that might be expected. The  $ET_o$  prediction is highly sensitive to the values of maximum temperature, meaning that accurate data for this factor or variable is important. Similarly, the relative humidity is the second most and sunshine duration is the third most important factor. Wind speed and minimum temperature are comparatively less important. So that the areas where wind speed data are not available, interpolation or gauging of the parameter from nearby stations, or use of historical average values may be used to estimate  $ET_o$  using the P–M method or the CROPWAT model.

In the context of calculating crop water requirements,

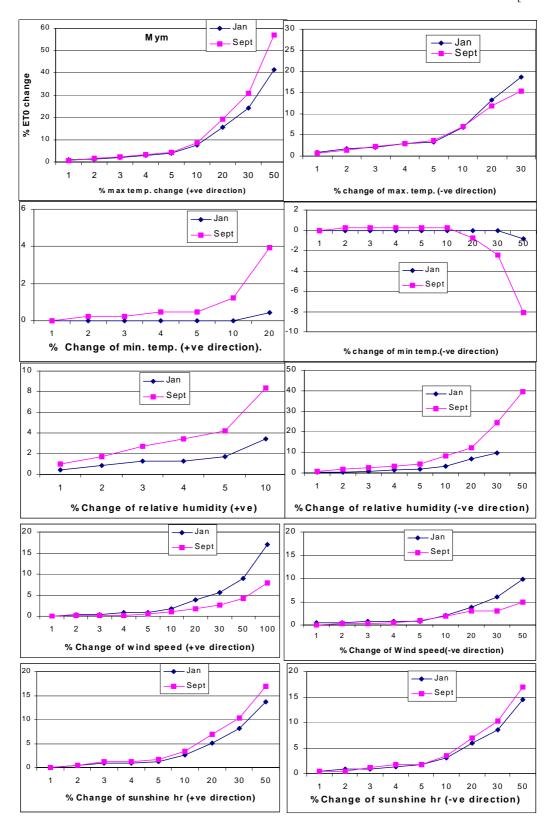


Fig.1a: Change of climatic variables (%) vs corresponding change in ET<sub>0</sub> values (%) (Mymensingh)

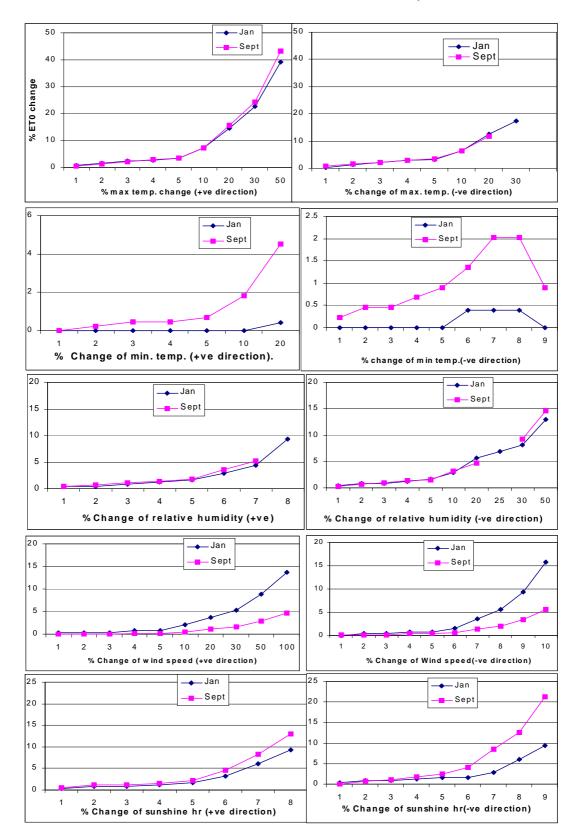
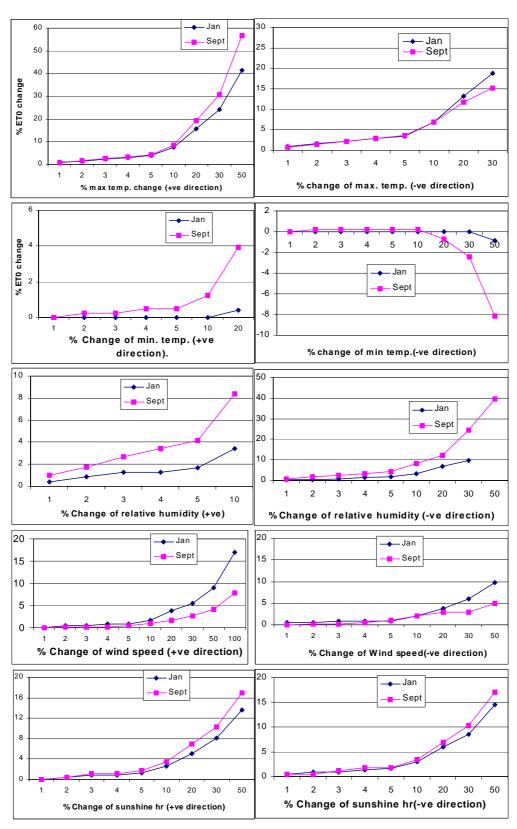


Fig. 1b: Change of climatic variables (%) vs corresponding change in ET<sub>0</sub> values (%) (Rajshahi)



**Fig.1c:** Change of climatic variables (%) vs corresponding change in ET<sub>o</sub> values (%) (Rangpur)

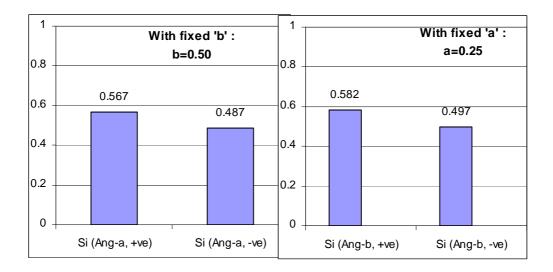


Fig. 2: Sensitivity of Angstrom coefficients (with the data of Mymensingh, dry period)

the accurate measurement of maximum temperature is particularly important. In selecting the appropriate method for  $ET_o$  estimation in a data short environment, the wind speed data may be interpolated or gauged from other locations or historical average values may used for the P-M method or CROPWAT model.

#### REFERENCES

Beven, K. (1979). Sensitivity analysis of the Penman-Monteith actual evapotranspiration estimates. *J. Hydrology*, 44: 169-190.

Coleman, R. D. and DeCoursey, D.G. (1976). Sensitivity and model variance applied to some evaporation and evapotranspiration models. *Water Resour. Res.*, 12:

873-879.

McCuen, R.H. (1974). A sensitivity and error analysis of procedures used for estimating evaporation. *Water Resour. Bull.*, 10: 486-498.

Piper, B.S. (1989). Sensitivity of Penmen estimates of evaporation to errors in input data. *Agric. Water Management*, 15: 279-300.

Smith, M., Allen, R., Monteith, J.L., Perrier, A., Santos Pereira, L. and Segeren, A. (1992). Expert Consultation on Revision of FAO Methodologies for Crop Water Requirements. Food and Agriculture Organization of the United Nations (Land and Water Development Division), Rome, 60 pp.

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