Journal of Agrometeorology 10(1) : 53 - 58 (June 2008)

Stochastic modelling of relative humidity at Banswara

S. R. BHAKAR, RAJ VIR SINGH, NEERAJ CHHAJED and ANIL KUMAR BANSAL
Deptt. of SWE, CTAE, Udaipur – 313 001 Rajasthan
E-mail : srbhakar@rediffmail.com

ABSTRACT

Study was conducted to develop stochastic model for monthly minimum and maximum relative humidity using 12 years (1992-2003) data of Banswara. The performed statistical test indicates that the series of monthly minimum and maximum relative humidity data are trend free. Their periodic components can be presented satisfactorily by the second harmonics. The stochastic components of both monthly minimum and maximum relative humidity follow second order Markov model. Validation of generated was made with measured series. A high correlation coefficient of 0.9980 and 0.9976 for mean monthly minimum and maximum relative humidity respectively was observed. The correlation was tested by t-test and found to be highly significant at 1 per cent level. The standard error is quite low. The regression equation was very close to 1:1 line. Therefore, the developed model could be used for future prediction of mean monthly minimum and maximum relative humidity, at Banswara.

Key words: Stochastic, auto correlation function, auto regression, relative humidity.

Minimum and maximum relative humidity are important weather parameters for estimation of crop water requirements (Allen *et al*., 1998; Bhakar, 2000; Bhakar and Singh, 2005). Frequently, it is required to estimate minimum and maximum relative humidity (RH) of places where measured data are not available. Study reveals that the process of change is so complex, that no exact laws have yet been established which can explain completely and precisely. Before such law can be established, simulation and mathematical modeling can approximate such complicated hydrologic system (Doob, 1953). In this way, the hydrologic system may be treated as the sum of periodic series and stochastic series. Periodic component takes into account the portion, which repeats after certain duration. The stochastic component constituted various random effects that cannot be estimated exactly. The stochastic analysis of time series is done to understand the mechanism that generate the data and to produce likely future sequence if required (Kottegoda, 1980).

MATERIALS AND METHODS

Location and data

The study was conducted fort the Agricultural Research Station, Banswara. The area comes under the Humid southern plain region of the agro-climatic zone IV-B of the state of Rajasthan, and is situated at 23° 33' N latitude, 74° 27' E longitude and at an altitude of 220 m above mean sea level. The annual rainfall in this region is 849.3 mm and more than 93% of this amount is received during the monsoon season alone, due to the influence of the southwest monsoon.

The minimum and maximum relative humidity data were collected from Agricultural Research Station, Banswara for 12 years (1992-2003). The mathematical procedure adopted for formulation of a predictive model based on stochastic component are the same as described in a companion paper Bhakar and Singh (2008).

The principal aim of the analysis is to obtain a reasonable model for estimating the generation process and its parameters by decomposing the original data series into its various components. Generally a time series can be decomposed into a deterministic component, which could be formulated in a manner that allowed exact prediction of its value, and a stochastic component, which is always present in the data and can not strictly be accounted for as it is made by random effects (Kottegoda, 1980). Standard procedure was followed as per guidelines of Kottegoda, 1980; Bhakar and Singh, 2005; and Jha *et al*., 2003.

RESULTS AND DISCUSSION

The statistical characteristics of the mean monthly

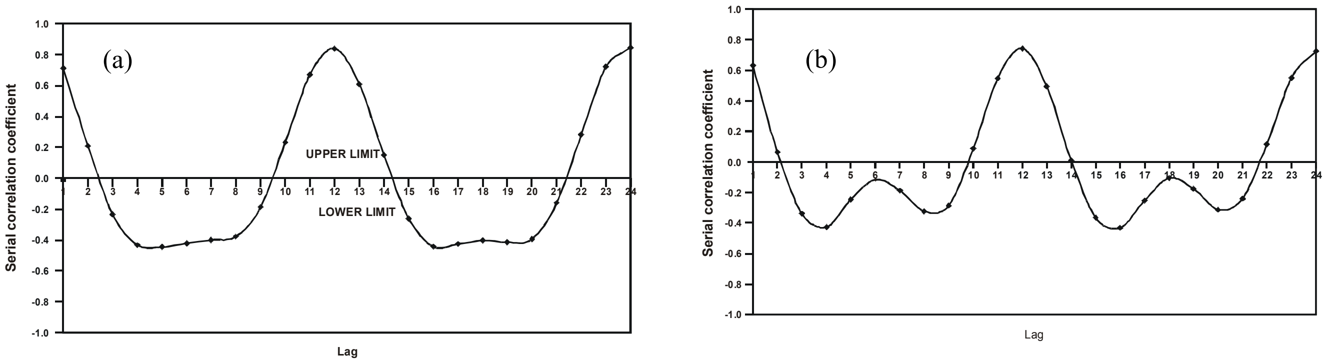


Fig.1: Correlogram of mean monthly (a) minimum and (b) maximum relative humidity

(a) (b)

Fig.2: Cumulative periodogram of mean monthly (a) minimum and (b) maximum relative humidity

minimum and maximum RH series were estimated. Data indicate that minimum RH values vary from 23 % in the month of March to 71 % in August while maximum RH values vary from 56 % in April to 89 % in August. Mean monthly minimum and maximum RH were found to be 39.71 % and 74.80 % respectively. Standard deviation, variance, kurtosis and skewness for mean monthly minimum relative humidity and mean monthly maximum relative humidity was determined. Variance in mean monthly minimum and maximum RH were found higher during month of July and January. This is due to change in season. Kurtosis is higher during July. This indicates maximum peak were observed during this period.

Serial correlation coefficient

The lag one serial correction coefficient of mean monthly minimum and maximum RH series were calculated and were found to be 0.710 and 0.634 respectively. The respective confidence limits were estimated as -0.209 to 0.189 (Kottegoda, 1980; Doorembos, 1976 and Faroda et al. 1999). The value of lag one serial correlation coefficients are outside the range of confidence limits and are significantly different from zero. This again confirms that RH series are time variant and not an independent one. Thus, both series may be modeled on stochastic theory. The mutual dependence of the observed mean monthly minimum and maximum RH series were also confirmed by the correlogram (Fig. 1).

Trend component

For detection of trend, the hypothesis of no trend was made and value of test statistics (z) was calculated by Turning Point Test and Kendall's Rank Correlation Test. The estimated value of test statistics (z) obtained for Turning Point Test and Kendall's Rank Correlation Test were within the 1% level of significance for both annual minimum and maximum RH series. Hence the hypothesis of no trend was accepted.

Further from the turning point test total number of turning points in annual minimum and maximum RH series were found to be 5 and 7 respectively. This indicates that both the series are random. From the above analysis it is confirmed that the observed series may be treated as trend free series.

Periodic component

Correlograms were drawn to confirm the presence of periodic component in mean monthly minimum and maximum RH series. The oscillation shape of the correlograms of the mean monthly minimum and maximum RH series confirm the presence of periodic component in both the series (Fig. 1). Further for both the series, the correlograms had peaks at legs equal to 12 and at other multiples of it. The time span of periodicity was taken as 12 for use in harmonic analysis of periodic component.

Determination of significant harmonics

For representing the periodic component of the mean monthly minimum and maximum RH series, the number of significant harmonics was determined by analyzing the cumulative periodogram. Only first two harmonics for both series, mean monthly minimum and maximum RH, were found highly significant. Other harmonics were not significant and therefore were ignored.

Parameters of periodic component

The Fourier coefficient A_k and B_k were estimated. The amplitude, phase angle and explained variance for different harmonics were calculated. These Fourier decompositions for mean monthly minimum and maximum RH series reveal that first two harmonics explained more than 83 and 80 per cent of variance respectively.

Cumulative periodogram test

A graph was drawn between P_i and number of harmonics. (Fig. 2) From this it can be observed that the first two harmonics appeared to be the periodic part of the fast increase and after that periodogram remains almost constant which may be treated as non - significant. The two criteria used to identify the number of significant harmonics to be used in modelling periodic component were found to be consistent, so first two harmonics were treated as significant and the remaining harmonics were ignored.

For the first two harmonics of the mean monthly minimum RH series, the values of Fourier coefficients (A1, A2 and B1, B2) were found to be -9.511, -1.771 and -16.688, 11.016 respectively. With these coefficients and the periodic component (P_t) were determined.

For the first three harmonics of the mean monthly maximum RH series the values of Fourier coefficients (A1, A2, and B1, B2) were found to be 1.559, -1.995 and -9.635, 8.378 respectively. With these coefficients and the periodic component (P_t) were determined.

Stochastic Component

After estimating the periodic component, a new stochastic time series was formed by subtracting the periodic component from historical time series.

Estimation of autoregressive parameters

The autoregressive parameters were estimated. The estimated values of auto covariance function and SCC of different lags (I_{max} = 24) for mean monthly minimum and maximum RH indicated the linear dependence between each lag. It is also revealed that the values of SCC are significantly different from zero in both the series which confirmed the dependence of the present values and the past values. In other words, it may be concluded that the past and present values are highly intercorrelated.

Selection of model order

Residual variance method was used to determine the order of the model which might significantly represent the non-deterministic stationary stochastic component. Residual variance at different lags was computed. The estimated values for 24 lags revealed that the minimum residual variance was obtained for order two for both mean monthly minimum RH and maximum RH series.

The residual series of stochastic component

The residual series (a_t) which is random independent part of stochastic component was obtained after removing the periodic and dependent stochastic parts from the historical series. The statistical analysis of the residual series confirmed its normal distribution

Table 1: Statistical parameters of the observed, generated and residual series of monthly minimum and maximum relative humidity

(a) (b)

Fig. 3: Correlogram up to lag 24 for residual series of monthly (a) minimum and (b) maximum relative humidity at Banswara

with mean which is almost equal to zero (mean = 0.0 and SD = 3.835 for mean monthly minimum RH series and mean = 0.0 and SD = 3.429 for mean monthly maximum RH series). The mean, SD of the historical and generated series are almost same for both the series which show closeness between historical and generated data (Table 1).

Model structure

The mathematical structure of the additive model could be represented as follows:

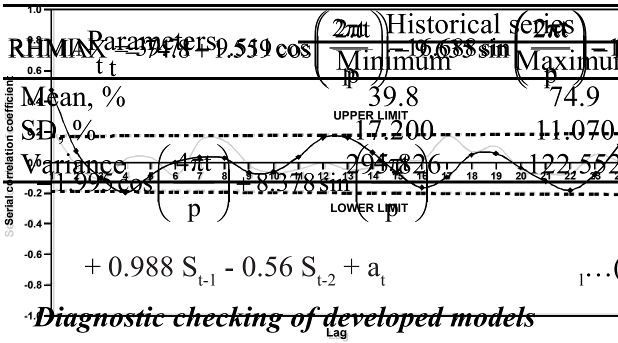
For mean monthly minimum RH series –

$$-1.771\cos\left(\frac{4\pi t}{p}\right)+11.016\sin\left(\frac{4\pi t}{p}\right)$$

$$+ 1.13\ S_{t-1} - 0.592\ S_{t-2} + a_t \qquad \ldots(1A)$$

and for mean monthly maximum RH series-

Parameters	Historical series		Generated series
	Minimum	Maximum	Minimum
Mean, %	39.8	74.9	39.8
SD, %	17.200	11.070	17.379
Variance	295.826	122.552	302.014



$$RHMAX_t = 74.8 + 9.519\cos\left(\frac{2\pi t}{p}\right) - 16.638\sin\left(\frac{2\pi t}{p}\right)$$

$$-1.995\cos\left(\frac{4\pi t}{p}\right) + 8.378\sin\left(\frac{4\pi t}{p}\right)$$

$$+ 0.988\ S_{t-1} - 0.56\ S_{t-2} + a_t \qquad \ldots(1B)$$

Diagnostic checking of developed models

The residuals obtained after fitting the formulated models were subjected to various analysis to test their adequacy for representing the time dependent structure of the mean monthly minimum and maximum RH series.

Sum of square analysis

The sum of squares of residual series was compared with sum of square of deviation of observed values from their mean. The value of coefficient of determination (R^2) was found to be 0.9503 and 0.9035 for minimum and maximum RH series respectively, which is nearly equal to unity. Thus, it could be concluded that the developed models have a fair

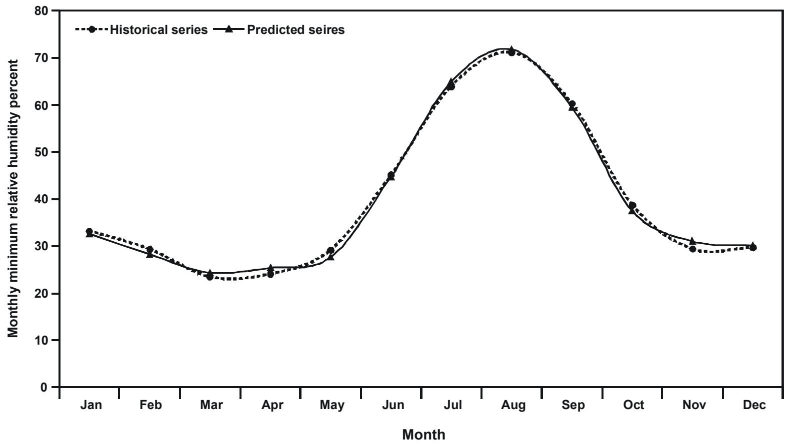


Fig. 4: Variation of generated and measured mean monthly minimum relative humidity

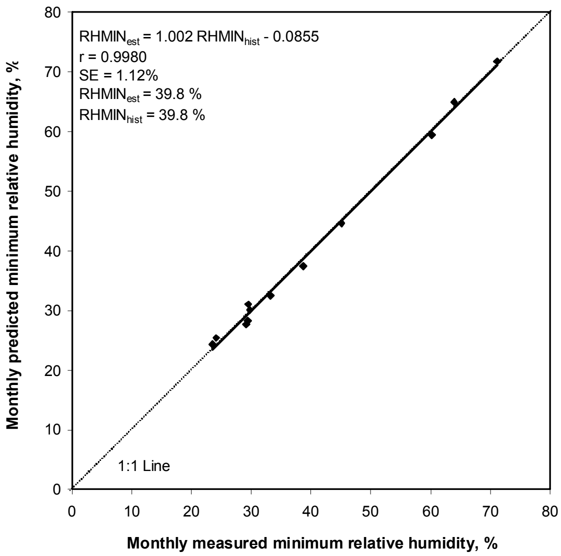


Fig. 5: Relationship between generated ($RHMIN_{est}$) and measured mean monthly minimum relative humidity ($RHMIN_{hist}$)

goodness of fit to the generated and measured mean monthly RH series.

Serial correlation analysis

The serial correlation coefficients (SCC) for lags l (l = 1, 2, 3….24) were computed for the residual monthly RH series. The values of SCC against respective lags were then plotted to obtain a correlogram. The resulting correlograms are shown in Fig. 3 with confidence limit at 1 per cent level for both the residual series. The correlograms are almost completely contained within the confidence limits at 1 per cent level. Hence it may be treated to be non significant. This confirms that the residual series may be treated as random series.

The residual series of mean monthly minimum RH series has a mean value of zero and the variance of 14.706 and the residual series of mean monthly maximum RH series have a mean value of zero and the variance of 11.824. This leads to the conclusion that the residuals are independent and normally distributed. It also confirms the randomness of the residuals.

Validation of stochastic models

Validation of generated 10 years mean monthly minimum RH series was made with 10 year mean monthly measured minimum RH series (Fig. 4) Which gave a correlation coefficient of 0.9980. The correlation was tested by t test and found to be highly significant at 1 per cent level. The standard error (1.12 %) is quite low. The estimated mean monthly minimum RH was found to be 39.8 %. Mean of the measured series was found to be 39.8 %. The regression equation is very near to 1:1 line (Fig. 5). Therefore, this model could be used for future prediction of monthly minimum RH.

Almost similar results with some numerical difference in values were obtained for the generated and historical mean monthly maximum series.

REFERENCES

Allen, R.G., Pereira, L.S. , Raes, D. and Smith, M. (1998). Crop Evapotranspiration, Guidelines for Computing Crop Water Requirement. FAO Irrigation and Drainage Paper 56. FAO Rome, Italy, P. 300.

Bhakar, S. R. (2000). Modelling of evaporation and evapotranspiration under climatic conditions of Udaipur. Ph. D. thesis submitted to faculty of Agricultural Engineering, Department of Soil and Water Engineering, CTAE, MPUAT, Udaipur.

Bhakar, S. R., and Singh, R.V. (2005). Stochastic modelling of minimum and maximum air temperature for Udaipur. Proceedings of 39th ISAE Annual Convention and Symposium. ANGR Agricultural University Hyderabad. Sec I HR 47: p 128

Bhakar, S. R. and Singh, R. V. (2008). Forecasting monthly wind speed for Udaipur region. *J. Agrometeorol.* 10(2):48-54

Doob, J.L. (1953). Stochastic Process. Wiley. New York.

Doorembos, J. (1976). Agro-meteorological Field Stations. Irrig and Drainage Paper -27. FAO Rome.

Faroda, A. S., Joshi, D.C. and Ram, B., (1999). Agroecological zones of north west hot arid region of India. *Annals Arid Zones,* **38**: 1-8.

Kottegoda, N.T. (1980). Stochastic Water Resource Technology. The Mac Millon Press Ltd., London

Jha, V., Singh, R. V. and Bhakar, S. R. (2003). Stochastic modelling of soil moisture. *J. Agric. Engi.,* **40(4)**: 51-56.

Received : November 2005; Accepted : December 2007