Weekly rainfall probability analysis by gamma distribution and artificial neural network

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ABSTRACT

Gamma distribution model (GDM) and artificial neural network (ANN) have been used to predict the weekly rainfall probabilities of Anand station of Gujarat, India using 48 years of rainfall data series (1958 to 2005). Estimated probabilities by GDM were compared with actual probabilities. Artificial neural network was used with back propagation algorithm and it was trained with the probabilities (%) obtained by GDM for weekly rainfall of 0.25cm and 0.5cm. Parameters used to train the neural network were number of hidden neurons 140, momentum 0.5 and error goal (computer error) $10^{-22}$. Probabilities obtained by ANN for different amounts of weekly rainfall were compared with probabilities obtained by GDM. The probabilities computed by both the methods GDM and ANN for getting certain fixed amount of rainfall were significant to actual probabilities. All the related programmes were developed in MATLAB.

Key words: Artificial neural networks, Gamma distribution

Rainfall is a discrete variable in any part of the India. Rainfall is very unevenly distributed in space and time since it may be excessive in one part of the country and deficient in another (Rao, 1976). Thus distribution of rainfall in India is erratic and its behavior unpredictable.

Many agricultural operations like crop sowing, crop harvesting, and pest control required daily or weekly probabilities rather than the information on average of rainfall. Gupta et al. (1975) have found the rainfall probabilities for agricultural planning. Suitable model is required to know the daily (Coe et al., 1982) or weekly rainfall probabilities (Biswas and Basarkar, 1982; Gabriel and Neuman 1962; Sarkar 2002; Victor and Sastry 1979). Mooley and Appa Rao (1973) and Khambete and Biswas (1978) showed that rainfall period less than four pentads (sum of five days rainfall) do not follow normal distribution and so used the incomplete Gamma Distribution to pentad rainfall of two stations in Rajasthan. Hargreaves (1975) and Sarker et al. (1982) have calculated the rainfall amount by Gamma distribution to obtain decade (sum of ten days rainfall) probabilities. To evaluate agricultural potential in Sudano-Sahelian
zone of West Africa, Davy et al. (1976) has used Gamma Distribution Model (GDM).

**MATERIAL AND METHODS**

The 48 years (1958-2005) of weekly rainfall data of Anand station has been used in the present study. The rainfall data of standard weeks from 22\textsuperscript{nd} (28\textsuperscript{th} May-3\textsuperscript{rd} June) to 42\textsuperscript{nd} (15\textsuperscript{th} -21\textsuperscript{st} October) were analysed to compute rainfall probability by different methods.

**Actual probabilities**

The probability of getting rainfall ‘p’ is computed by actual occurred weekly rainfall during 48 years period. The probabilities of getting x amount of rainfall (P'x (x)), was computed using MATLAB function namely ‘gammainc’ and modified by Kulshrestha et al (2000) as P\textsubscript{x} (x) = p + 1-p) P' x (x)

Here probabilities of getting zero rainfall (p) were also considered during the computation of probabilities.

**Gamma distribution model (GDM)**

Weekly mean rainfall in the summer monsoon period is always positive (Fig.1). Rainfall amount varies between 0 to x. Therefore GDM is applicable as its cumulative distribution function is given by

$$P\textsubscript{x} (x) = \int_{0}^{x} \frac{\lambda^{\eta} x^{\eta-1} e^{-\lambda x}}{\Gamma(\eta)} \, dt$$

Where, scale (\(\lambda\)) and shape (\(\eta\)) parameters were obtained by weekly mean rainfall data following the method of maximum likelihood (Han,1977).

Scale parameter (\(\lambda\)) showed decreasing trend with linear fit

$$\lambda = -0.0003302x + 0.0026617$$

to weekly rainfall with residual of 0.0027827 cm. and fitting a cubic equation,

$$\lambda = -3.9806e-005 x^3 + 0.00062221 x^2 - 0.0032241 x + 0.0065392$$

residual of 0.0016246 cm (Fig.2).

Shape parameter (\(\eta\)) showed increasing trend with linear fit

$$\eta = 0.00096386 x + 0.041577$$

to weekly with residual of 0.065048 cm and fitting of cubic equation,

$$\eta = 0.0007462 x^3 + 0.0095099 x^2 - 0.034617 x + 0.079563$$

had residual of 0.056484 cm (Fig.3). The probability computed by GDM were compared with that obtained actual probabilities for different amounts of rainfall.

**Artificial neural network (ANN)**

An artificial neural network (ANN) is a flexible mathematical structure, which is capable of identifying complex non-linear relationship between input and output data set. ANN models have been found useful and efficient, particularly in problems for which the characteristics of the processes are difficult to describe using mathematical equations (Zaldívar et al., 2000). ANN
provides input-output simulation and forecasting models in situations that do not require modeling of the internal structure of the parameters.

ANN was used with back propagation algorithm. It trained with the probabilities (%) of weekly rainfall of 0.25 cm and 0.5 cm obtained by GDM. Table 1 shows the parameters used to train the ANN, considering number of hidden neurons is 140 with momentum 0.5 with error goal (computer error) $10^{-22}$. Here, ANN consists of three layers. The input, the hidden and the output layer (Fig. 4). Input and output layer have one neuron. Here input is weekly
Fig. 3: Relation between shape parameter and weekly rainfall

Fig. 4: Actual and GDM (*) weekly probabilities at 1.25 cm rainfall

rainfall and output is probabilities for the standard weeks of 22\textsuperscript{nd} to 42\textsuperscript{nd}. Probabilities are obtained for 0.5, 7.5, 12.5, 17.5, 25 and 37.5 cm of weekly rainfall and compared with probabilities obtained by GDM.

RESULTS AND DISCUSSION

\textit{Gamma distribution model}

Probabilities of getting weekly rainfall of 1.25 cm computed by GDM are compared with actual probabilities (Fig.4).
As the monsoon advances the probabilities increased. Differences of actual and computed probabilities by GDM showed highest difference of 12% during the standard week 33. Rest of the differences were less than 7%.

Fig. 5 shows the predicted weekly rainfall at different probability levels 10 to 100%. As the probabilities increased weekly rainfall decreased. For the 100% probabilities, weekly rainfall was non-zero, small number. Computed probabilities were tested with student t-test for two tails and were found significant with actual probabilities.

Artificial neural network (ANN)
Table 1: Details of the parameter values used in ANN training

<table>
<thead>
<tr>
<th>Sr. No.</th>
<th>Rainfall (cm)</th>
<th>Number of epochs used</th>
<th>Learning rate</th>
<th>Moment Tum.</th>
<th>No. of neurons</th>
<th>Error goal</th>
<th>RMSE Related by GDM</th>
<th>PAE (%) Related by GDM</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.25</td>
<td>14772</td>
<td>0.001</td>
<td>0.5</td>
<td>140</td>
<td>$10^{-22}$</td>
<td>4.035</td>
<td>8.57</td>
</tr>
<tr>
<td>2</td>
<td>2.5</td>
<td>14772</td>
<td>0.001</td>
<td>0.5</td>
<td>140</td>
<td>$10^{-22}$</td>
<td>2.82</td>
<td>8.56</td>
</tr>
</tbody>
</table>

Computed probabilities by ANN for weekly rainfall of 1.25 and 2.5 cm (Fig. 6) were compared with obtained probabilities by GDM. Computed probabilities by ANN were found significant with GDM probabilities by student t-test. Other probabilities predicted by ANN of getting weekly rainfall of 5.0, 7.5, 12.5, 17.5, 25 and 37.5 cm were also found significant to probabilities obtained by GDM. Root mean square error (RMSE) and Percentage of average error (PAE) during the use of ANN were less than 10% (Table 1).

The probabilities computed by ANN were significant to probabilities found by GDM and GDM probabilities were significant to the actual probabilities. Therefore, it was concluded that probabilities by ANN were also significant to actual probabilities.

REFERENCES


Hargreaves, G.H. 1975. Water requirement manual for irrigated crops and rainfed agriculture. EMBRAPA and Utah


